A proposal for manifold learning

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This is work in progress, that might have applications in diverse branches of science. The original stimulus was the examination of images from molecular medicine. Here is a statistical model for the examination of data gathered experimentally with stochastic errors. When this particular model is applicable, we extract (approximate) piecewise polynomial formulas governing the distribution of the data.

Let M be a smooth compact manifold of dimension m smoothly embedded in $\mathbb{R}^n$. Then M inherits a Riemannian structure, and hence a Riemannian measure on M. Let $Y = (y_1, \dots, y_N) \subset M$ be randomly sampled from this measure. Let $U = (u_1, \dots, u_N) \subset \mathbb{R}^n$ be a random sample from the standard normal distribution $\mathcal{N}_\varepsilon$ centred at 0, with variance $\varepsilon^2.I_n$, where I_n is the unit $n \times n$-matrix. Let $X = (x_, \dots, x_N) \subset \mathbb{R}^n$, where $x_i = y_i + u_i$. Now suppose X is given, but m, M and ε are unknown. The problem is to recover an (approximation to) M from X.