Abstracts

Denis Bernard CNRS/ENS-Paris
Christian Hagendorf University of Virginia

The Gaussian free field and SLE(4) on doubly connected domains

In this talk we consider the Gaussian free field whose level lines are known to be related to SLE(4). It is shown how this relation allows to define chordal SLE(4) processes on doubly connected domains, describing traces that are anchored on one of the two boundary components. The precise nature of the processes depends on the conformally invariant boundary conditions imposed on the second boundary component. Starting from the free field theory I will show how to generalize Schramm's formula for left-right-passage with respect to a point to doubly connected domains for different boundary conditions, and point out a relation to first-exit problems for Brownian bridges. For the free field compactified at the self-dual radius, the extended $su(2)_1$ symmetry leads to a class of conformally invariant boundary conditions parametrized by elements of $SU(2)$. I will explain how to extend SLE(4) to this setting. This allows for a derivation of new passage probabilities à la Schramm that interpolate continuously from Dirichlet to Neumann conditions.

John Cardy
Oxford University

2D Field Theory and Random Planar Sets: Past and Future

I will give an overview for non-specialists of the connections between field theory and rigorous approaches to studying the scaling limits of random planar objects, focussing on past successes (e.g. deriving CFT conjectures from SLE and other direct approaches) and future challenges (for example other CFT conjectures, multiply connected domains, and results concerning off-critical scaling limits.)

Béatrice de Tilière
University of Neuchâtel

The critical Z-invariant Ising model via dimers.

Fisher established an explicit correspondence between the Ising model and the dimer model on a decorated graph. In collaboration with Cedric Boutillier, we solve fundamental aspects of the dimer model corresponding to the Ising model at criticality, by proving explicit formulas for the free energy, and for local statistics. Moreover, we show that these expressions have the surprising feature of depending only on the local geometry of the underlying graph.

Benjamin Doyon
Kingś College London

Conformal field theory from conformal loop ensembles

Conformal loop ensembles (CLE) provide (sometimes conjecturally, sometimes provably) a probability theory for the scaling or "continuum" limits of critical models, through random sets of non-intersecting loops. These scaling limits are also believed to be described by conformal field theory (CFT), which rather uses algebraic and analytic ideas. I will overview some of my works on relating the two theories. They are based on the CLE construction of the stress-energy tensor, the most fundamental quantum field in CFT. After an introduction to scaling limits, CFT and CLE, I will briefly explain what the stress-energy tensor is in CFT in the language of conformal derivatives (a particular type of Hadamard derivatives), and I will describe how this characterisation allows us to identify the stress-energy tensor as a certain (limit of a) random variable in CLE, based on the main axioms and general properties of CLE. This provides the conformal Ward identities, Cardy's boundary conditions, as well as the correct transformation properties. If time permits, I will also discuss some recent ideas about reproducing in CLE the full identity sector of CFT, and its Virasoro vertex operator algebra structure.

Julien Dubedat
Columbia University

Dimers and analytic torsion

We discuss Gaussian invariance principles for dimer models in relation with variational formulae for zeta-determinants of Cauchy-Riemann operators.

Hugo Duminil-Copin
University of Geneva

Self-avoiding walk on the hexagonal lattice

We will prove a conjecture made by B. Nienhuis regarding the connective constant of the hexagonal lattice. More precisely, we will show that the number a_n of self-avoiding walks of length n (starting at the origin) satisfies:

$$\lim_{n\to\infty} \frac{1}{n} \log a_n = \sqrt{2+\sqrt{2}}.$$

The proof uses a parafermionic observable for the self avoiding walk, which satisfies a half of the discrete Cauchy-Riemann relations. Establishing the other half of the relations (which conjecturally holds in the scaling limit) would also imply convergence of the self-avoiding walk to SLE(8/3). This is a joint work with S. Smirnov.

Bertrand Duplantier
Institute for Theoretical Physics, Saclay

Perspectives on Liouville Quantum Gravity and KPZ

Polyakov first understood in 1981 that the summation over random Riemannian metrics involved in transition amplitudes in gauge and string theories could be represented mathematically by the now celebrated Liouville theory of quantum gravity. The quantum gravity measure is formally defined by $d\mu = e^{\gamma h(z)}dz$, where dz is the 2D Euclidean (i.e., Lebesgue) measure; $e^{\gamma h(z)}$ is the random conformal factor of the Riemannian metric, with h an instance of the Gaussian free field (GFF) on a bounded domain D; and γ a constant, $0 \leq \gamma < 2$.

In 1988, Knizhnik, Polyakov and Zamolodchikov (KPZ) predicted that corresponding critical exponents (x) of a conformally invariant statistical model in the Euclidean plane and in Liouville quantum gravity (Δ) would obey a quadratic relation depending on γ.

A probabilistic and geometrical proof of this relation uses a properly regularized quantum measure, defined in terms of the mean value on successive circles centered at z of GFF h. This measure has both a limit, the Liouville quantum measure, and a Brownian representation, of which the KPZ relation appears as a martingale or large deviations property.

The singular case $\gamma > 2$ corresponds to a critical proliferation of bubbles ("baby-universes") and is related to the regular one for $\gamma' < 2$ by the fundamental duality $\gamma\gamma' = 4$.

Open problems and future directions include the relation of Liouville quantum gravity to discrete random lattices and statistical models and their critical continuum limit, in particular the description of geodesics and the coupling to Stochastic Schramm-Loewner Evolution.

This is joint work with Scott Sheffield.

Tom Ellis
University of Cambridge

From Diffusion Limited Aggregation to the Browian Web via Conformal Mappings

The Hastings-Levitov (HL) models are a one-parameter family of models of random growth processes inspired by "Diffusion Limited Aggregation". A growing cluster is represented by an iterated sequence of conformal maps. I will show how to relate (in a suitable scaling limit where the size of the arriving particles approaches zero) the harmonic measure on the surface of the growing HL(0) cluster to a system of coalescing Brownian motions.

Ilya Gruzberg
University of Chicago

Quantum Hall transitions and conformal restriction

Disordered electronic systems exhibit continuous quantum phase transitions between insulating and conducting phases (Anderson transitions). The nature of the critical state at and the critical phenomena near such a transition are of great current interest. While superficially similar to transitions in clean statistical mechanical systems (such as the Ising model, etc.), Anderson transitions exhibit many features specific to disordered systems. A famous example is the integer quantum Hall (IQH) plateau transition. In spite of much effort over several decades, an analytical treatment of most of the critical states in disordered electronic systems has been elusive. Inspired by recent progress in rigorous understanding of critical phenomena and conformal invariance in two dimensions, we propose to use the recently developed rigorous theory of conformal restriction and Schramm-Loewner evolutions to study the IQH and other Anderson transitions in 2D. We consider the so-called point contact conductances (PCC) and obtain, for the first time, exact analytical results for PCC's in the presence of a variety of boundary conditions at the IQH and similar critical points.

Clément Hongler
University of Geneva

Ising interfaces with free boundary conditions

We study the Ising model at criticality from an SLE point of view.

The interfaces between + and − spins of the Ising model with Dobrushin +/− boundary conditions have been shown to converge to SLE(3) by Chelkak and Smirnov, thanks to the introduction and proof of convergence of a discrete holomorphic martingale observable in this setup.

We show conformal invariance of the Ising interfaces in presence of free boundary conditions. In particular we prove the conjecture of Bauer, Bernard and Houdayer about the scaling limit of interfaces arising in a so-called dipolar setup. The limiting process is a Loewner chain guided by a drifted Brownian motion, known as dipolar SLE or SLE$(3, -3/2)$ in the literature.

This case is made harder by the absence of natural discrete holomorphic martingales, requiring us to introduce "exotic" martingale observables. The study of these observables is allowed by Kramers-Wannier duality and Edwards-Sokal coupling, and the computation of the scaling limit is made by appealing to discrete complex analysis methods, to three existing convergence results about discrete fermions, to the scaling limit of critical Fortuin-Kasteleyn model interfaces and to the introduction of Coulomb-gases integrals.

Our result allows to show early predictions by Langlands, Lewis and Saint-Aubin about conformal invariance of crossing probabilities for the Ising model.

This is based on joint work with Kalle Kytölä and work in progress with Hugo Duminil-Copin.

Yacine Ikhlef
University of Geneva

Discrete parafermions and Coulomb Gas in the square O(n) model

The square-lattice O(n) model has several integrable lines, not all corresponding to a simple Coulomb Gas. We discovered recently the presence of discrete holomorphic parafermions in this model, which generally lead to an SLE description of the interfaces. This motivates a more profound study of the Bethe-Ansatz solution, to determine the continuum degrees of freedom and the precise constraints on their excitation sectors.

Konstantin Izyurov **Kalle Kytölä**
University of Geneva

Hadamard's formula and couplings of SLE with GFF

Given a Gaussian Free Field in a planar domain with some boundary conditions (e.g. Dirichlet, Neumann, Riemann-Hilbert etc.), we provide a framework to compute drifts of SLE-type processes describing level lines of this field. This leads to a number of interesting examples. We consider natural GFF variants in the annulus that are coupled with standard annulus SLE, various annulus analogues of chordal SLE(κ, ρ), and chordal SLE-type curves from outer boundary to the inner one with prescribed winding. Some of the examples coincide with those recently considered by C. Hagendorf, M. Bauer and D. Bernard, but our technique is different. In all cases, the drift term is computed explicitly, and the existence of the coupling is proved. Our methods apply to the multiply-connected case.

Nam-Gyu Kang
Seoul National University

Radial Conformal Field Theory

I will present conformal field theory (radial CFT) in a simply connected hyperbolic Riemann surface with a marked interior point and a marked boundary point. Two types of radial CFT are introduced to show their relations to the radial SLEs.

I will focus on presenting twisted radial CFT generated by the free field with Z_2-twisted boundary conditions. This CFT is closely related to the construction of radial harmonic explorer introduced by N. Makarov and D. Zhan. This is joint work with N. Makarov

Antti Kemppainen
Université Paris-Sud XI

Random curves, scaling limits and Loewner evolutions

In the 2D statistical physics and its lattice models, interfaces are random curves. A general strategy to prove the convergence of a random discrete curve, as the lattice mesh goes to zero, is first to establish precompactness of the law giving the existence of subsequential scaling limits and then to prove the uniqueness. In this talk, I will introduce a sufficient condition that guarantees the precompactness and also that the limits are Loewner evolutions, i.e. they correspond to continuous Loewner driving processes. This framework of estimates is applicable in almost all proofs aiming to establish that an interface converges to a Schramm-Loewner evolution (SLE). In principle, it can be applied beyond SLE. This is joint work with S. Smirnov.

Rick Kenyon
Brown University

Dimers

An expository talk about the dimer model and its connections to random interfaces and conformal invariance.

Peter Kleban
University of Maine

Factorization of density correlation functions for clusters touching the sides of a rectangle

We consider the density, at a point $z = x + iy$, of clusters that touch the left ($P_L(z)$), right ($P_R(z)$), or both ($P_{LR}(z)$) sides of a rectangular system, with wired (open) boundary conditions on the left and right (top and bottom) sides, at the critical point of the Q-state Potts models for $0 \leq Q \leq 4$. While each of these quantities is nonuniversal and indeed vanishes in the continuum limit, the ratio

$$C(z) = \frac{P_{LR}(z)}{\sqrt{P_L(z)\, P_R(z)\, \Pi_h}} , \tag{1}$$

is a universal function of z. Here for percolation ($Q = 1$) Π_h is the probability of left-right crossing given by Cardy, suitably generalized for other Q values. We evaluate these quantities using conformal field theory, calculating the corresponding six-point correlation functions. As a result, we find that $C(z)$ depends upon x but not upon y.

These correlation functions, aside from algebraic pre-factors that cancel out of the ratio C, depend on three cross-ratios, one of which determines the aspect ratio of the rectangle. Parameterizing the conformal map to the half-plane is crucial; with the proper choice (for a given aspect ratio) the correlation functions, aside from pre-factors, only depend on one variable, resulting in the y independence.

For percolation, for instance, $C(z)$ goes to a constant

$$C_0 = \frac{2^{7/2}\, \pi^{5/2}}{3^{3/4}\, \Gamma(1/3)^{9/2}} = 1.0299268\ldots , \tag{2}$$

for points far from the ends, and varies by no more than a few percent for all z values. Thus $P_{LR}(z)$ factorizes over the entire rectangle to very good approximation. High-precision numerical simulations verify these results.

Joint work with Jacob J. H. Simmons and Robert M. Ziff.

Michael Kozdron
Unievrsity of Regina

A rate of convergence for loop-erased random walk to SLE(2)

Among the open problems for SLE suggested by Oded Schramm in his 2006 ICM talk is that of obtaining reasonable estimates for the speed of convergence of the discrete processes which are known to converge to SLE. In this talk we derive a rate for the convergence of the Loewner driving function for loop-erased random walk to Brownian motion with speed 2 on the unit circle, the Loewner driving function for radial SLE(2). This talk is based on joint work with Christian Benes (CUNY) and Fredrik Johansson (KTH).

Antti Kupiainen
Helsinki university

Random Conformal Welding

We explain a construction of a conformally invariant random family of closed curves in the plane by welding of random homeomorphisms of the unit circle. The random homeomorphism is constructed using the exponential of the restriction of the two dimensional free field on the circle.

Joint work with Kari Astala, Peter Jones, Eero Saksman

Gregory F. Lawler
University of Chicago

The SLE curve - a review and a look to the future

This will be a review talk starting with the definition of the Schramm-Loewner evolution (SLE) and discussing what we know about the curve. Topics include: existence of curve and Holder continuity properties, fractal properties, Green's function, restriction and SLE in subdomains, multiple SLEs Girsanov and SLE(κ,ρ) processes, reversibility and duality, natural parameterization. I hope to discuss some recent areas of research and open problems.

Jean-Francois Le Gall
Universite Paris XI and Institut universitaire de France

The Brownian map : A continuous limit for large random planar maps.

Planar maps are graphs embedded in the plane, considered up to continuous deformation. They have been studied extensively in combinatorics, and they have also been used in theoretical physics, where they serve as models of random geometry. The problem of the existence of a scaling limit for large random planar maps viewed as random metric spaces was stated by Oded Schramm in his 2006 ICM paper, in the special case of triangulations. We discuss recent results related to this problem as well as remaining open problems. In the case of bipartite planar maps, one can show that sequential limits of rescaled random planar maps with a large number of faces are described by the so-called Brownian map, which is a quotient space of Aldous' Continuum Random Tree (the CRT) for an equivalence relation defined in terms of Brownian labels assigned to the vertices of the CRT. The Brownian map can be viewed as a "Brownian surface", in the same sense as Brownian motion is the limit of rescaled random walks with a large number of steps. Although the key problem of the uniqueness of the distribution of the Brownian map is still open, many of its properties can be investigated in detail. We discuss recent work concerning the uniqueness of geodesics in the Brownian map.

Yves Le Jan
Paris Sud 11

Loops, fields, and operators.

We present several relations between Markovian loops ensembles, free fields and associated operators in the context of discrete spaces and in the continuum limit.

Nikolai Makarov
Caltech

Random normal matrices

This will be an expository talk concerning the distribution of eigenvalues in random normal matrix ensembles. The theory of this Coulomb gas type model (electrons in an external field) has three closely connected and equally interesting levels –classical (Laplacian growth),statistical (or quantum), and field theoretical.

I will review recent mathematical developments in all three areas.A rich physical content of the theory has been discovered in a remarkable series of papers by P. Wiegmann, A. Zabrodin, et al.

The talk will be based on my joint work with Y. Ameur and H. Hedenmalm, and with N.-G. Kang.

Grégory Miermont
Universit de Paris-Sud 11

Scaling limits of random planar maps with large faces

We discuss asymptotics of large random maps in which the distribution of the degree of a typical face has a polynomial tail. When the number of vertices of the map goes to infinity, the appropriately rescaled distances from a base vertex can be described in terms of a new random process, defined in terms of a field of Brownian bridges over the so-called stable trees.

This allows to obtain weak convergence results in the Gromov-Hausdorff sense for these m̈aps with large faces;̈ viewed as metric spaces by endowing the set of their vertices with the graph distance. The limiting spaces form a one-parameter family of s̈table maps;̈ in a way parallel to the fact that the so-called Brownian map is the conjectured scaling limit for families of maps with faces-degrees having exponential tails. This work takes part of its motivation from the study of statistical physics models on random maps. Joint work with J.-F. Le Gall.

Jason Miller
Stanford University

Universality for SLE(4)

We resolve a conjecture of Sheffield that SLE(4) is the universal limit of the chordal zero-height contours of random surfaces with isotropic, uniformly convex potentials.

Bernard Nienhuis
Universiteit van Amsterdam

Tilings of irregular hexagons

A number of physical systems can be represented by tilings of the plane by hexagons of which the sides are irregular in length, but fixed in orientation. These forms are found in adsorbed monolayers of atoms on a hexagonal crystal surface. In a completely different context these tilings show up in a diagram of the forces between hard spheres packed in a hexagonal array. The number of ways to tile a triangular domain with hexagons with sides of integer length, plays a role in product formula of Schur functions.

We will show some properties of the model. It can be solved by means of the Bethe Ansatz. We consider an anisotropic tiling, in which the mean length of the sides is different for the three orientation. The partition integral can be expanded in powers of the shortest one.

Scott Sheffield
Department of Mathematics at MIT

Scaling limits of random planar maps

The critical random-graph FK models are a natural way of constructing random (discretized) loop-decorated two-dimensional manifolds (homeomorphic to the sphere).

We would like to show that the scaling limit of these surfaces is some kind of a continuum "random surface". The limiting object will be fractal and singular, not a manifold in the usual sense. Rather, it should be a random element of the completion—with respect to some topology—of the space of loop-decorated manifolds.

What topology should we use? The choice actually matters, as it determines the nature of the limiting object and the properties encoded by the limit. For example, if one uses a Gromov-Hausdorff topology, the limit should be a random metric space. If one uses the natural conformal topology, the limit should be a random measure on the sphere. In a certain third topology (the so-called driving function topology) the limit becomes a kind of Brownian motion closely related to Liouville quantum gravity and certain couplings between SLE and the Gaussian free field.

We discuss recent progress and open problems from each of these perspectives.

Misha Sodin
Tel Aviv University

Nodal lines of random waves. Many questions and few answers.

In the talk, I will introduce random plane and spherical waves, and will describe recent attempts to understand the misterious and beautiful structure of their nodal lines.

Wendelin Werner
Université Paris-Sud

SLE, loops, bubbles, trees and fields.

The goal of this talk will be to explain various ways to describe geometrically via SLE-type loops, SLE-trees or SLE-bubbles, the scaling limits of lattice models, or the Gaussian free field. I will try to survey what is known, and some of the main open questions in this direction.

P. Wiegmann
University of Chicago

Laplacian Growth, DLA and Analytic Geometry

Laplacian growth represents a broad class of growth phenomena where a planar domain grows with a rate equal to harmonic measure of its boundary. Hele-Shaw flow of a boundary of a driven viscous fluid is the most noticeable example in hydrodynamics. Diffusion Limit of Aggregation (DLA) is an example of a stochastic growth process govern by the same rule. It is realized through Brownian excursions of particles with a small size with absorbing growing boundary. Global statistical properties of DLA clusters are unknown.

Both processes are singular: a fluid boundary develops cusp-simgularities; the DLA produces a branching extended cluster with a width controlled of one particle, such that no continuum limit is possible.

Recent development on the fluid dynamics side of the problem shows that singularities trigger viscous shocks propagating through the viscous fluid. Shocks form a branching growing graph reminiscent to computer generated DLA patterns.

Viscous shocks have rich analytical structure. They have been identified with Stokes-level lines of the so-called Krichever-Boutroux complex curves. These very special curves appear in different (but apparently related) mathematical disciplines: asymptotes of orthogonal polynomials and semiclassical description of Painlevé transcendents. These relations place a problem of global properties of growth clusters to the domain of analytic geometry.

In the talk I will review

(i) a relation between hydrodynamics of Hele-Shaw flow and Stochastic growth problem of DLA. (ii) singularities in Hele-Shaw flow; (iii) Viscous shocks and Krichever-Boutroux curves; (iv) Painlevé transcendents as an integrable regularization of ill-defined non-linear equations of the Laplacian growth.

Dapeng Zhan
MIchigan state University

Reversibility of whole-plane SLE

Whole plane SLE is viewed as the limit of radial SLE if the target, say b, is fixed and the domain tends to the whole Riemann sphere without a single point, say a. It describes a random curve in the Riemann sphere that grows from a to b. The curve is simple if the parameter $\kappa \le 4$. In this talk I will explain my recent work: whole plane SLE satisfies reversibility if $\kappa \le 4$. The proof uses two tools: one is the stochastic coupling technique, which was used to prove the reversibility of chordal SLE when $\kappa \le 4$, and the duality of SLE; the other is the annulus Loewner equation, which was introduced to define SLE in doubly connected domains. The main idea of the proof is to grow a pair of whole plane SLE, one is from a to b, the other is from b to a, such that they are weakly independent before they meet, and every point on one curve is visited by the other curve, and so the two curves overlap. From this result, we see that the radial SLE curve near its target point behaves similarly to the whole plane SLE curve near its initial point.