Combinatorics, Dynamics, Probabilities

Swedish-Russian conference

3-7 October 2000

Stockholm, Royal Institute of Technology

Supported by the NFR (The Swedish Natural Science Research Council),

the KVA (The Royal Swedish Academy of Sciences),

and the RFBR (The Russian Foundation for Basic Research)

Organizers

Tatiana Smirnova-Nagnibeda, KTH. E-mail: `tatiana@math.kth.se`
Stanislav Smirnov, KTH. E-mail: `stas@math.kth.se` Fax: 46-8-7231788

Program

Tuesday, October 3, Kollegiesalen

Time	Title	Lecturer
8.30-9.00	Registration	
9.00	Opening	
9.20-10.10	Random matrices, random growth and random tilings (I)	Kurt Johansson KTH Stockholm
	coffee break	
10.40-11.30	Flows on surfaces and related geometric questions (I)	Dmitry Anosov Steklov Institute, Moscow
11.40-12.30	Brownian limits for combinatorial problems (I)	Svante Janson Uppsala University
	lunch	
14.00-14.30	The random assignment problem	Svante Linusson University of Linköping
14.40-15.00	Limit theorems for partitions	Yuri Yakubovich Steklov Institute, St.Petersburg
15.05-15.25	Topological entropy for random dynamical systems and the analog of Misiurewicz-Przytycki theorem	Alexei Fishkin Moscow State University
15.30-15.50	On smooth models of dynamical systems with discrete spectrum	Alexander Prikhod'ko Moscow State University
	coffee break	
16.20-16.40	Building the tree of life	Niklas Eriksen KTH Stockholm
16.45-17.05	Graphs with large girth	Maxim Vsemirnov Steklov Institute, St.Petersburg
17.15-17.35	A generalization of the law of large numbers for random matrices	Stefan Israelsson Uppsala University
17.40-18.00	The equivalence of notions of entropy for diadic sequences of partitions	Alexander Gorbulskij St.Petersburg State University

Wednesday, October 4, Kollegiesalen

Time	Title	Lecturer
9.00-9.50	Trends in percolation theory (I)	Olle Häggström, Chalmers UT and Göteborg University
10.00-10.50	Ergodicity and conservativity: random walks, Brownian motion, geometric flows (I)	Vadim Kaimanovich Université de Rennes, France
	coffee break	
11.20-11.50	Percolation diffusion	Torbjörn Lundh, Chalmers UT and Göteborg University
12.00-12.30	Explicit isoperimetric constants for planar graphs and applications to percolation theory	Johan Jonasson, Chalmers UT and Göteborg University
	lunch	
14.00-14.20	New applications of Bohnenblust-Spitzer combinatorial identity: joint distributions for random walks and a three-term Szego formula	Dimitri Gioev KTH Stockholm
14.25-14.45	Global bifurcations on the Klein bottle. Unimodal case	Anton Borisyuk Moscow State University
14.55-15.15	Geometry and analysis of de Rham's curve	Pavel Nikitin St.Petersburg State University
15.20-15.40	On a relation between time average and minimal attractors	Pavel Bachurin Moscow State University
	coffee break	
16.10-16.30	On invariant manifolds in singularly perturbed systems	Olga Anosova Moscow State University
16.35-16.55	Discrete quasi-periodic Schrödinger equations	Kristian Bjerklöv KTH Stockholm
17.05-17.25	An isotopic classification of spatial 3-valent graphs	Vitali Kurlin Moscow State University
17.30-17.50	On uniqueness of solution of Riemann-Hilbert problem: analytical classification of Fuchsian singular points	Victor Kleptsyn Moscow State University

Thursday, October 5, Kollegiesalen

Time	Title	Lecturer
9.00-9.50	Random matrices, random growth and random tilings (II)	Kurt Johansson
10.00-10.50	Brownian limits for combinatorial problems (II)	Svante Janson
	coffee break	
11.20-11.50	Schreier graphs of self-similar group actions on the Cantor set	Volodymyr Nekrashevich Kiev University, Ukraine
12.00-12.30	On the fine structure of stationary measures in IFS which contract-on-average	Nikita Sidorov U.Sc.Tech. in Manchester, UK
12.40-13.00	Orbits of Zariski dense semigroups of SL(n,Z)	Roman Muchnik Yale University, USA

Afternoon free

Friday, October 6, Kollegiesalen

Time	Title	Lecturer
9.00-9.50	Flows on surfaces and related geometric questions (II)	Dmitry Anosov
	coffee break	
10.20-11.10	The problem of reversibility and polymorphisms which are quasisimilar to automorphisms	Anatoly Vershik Steklov Institute, St.Petersburg
11.20-12.30	Problem session	
	lunch	
14.00-14.20	Euler characteristic of the space of real meromorphic functions	Sergei Shadrin Moscow Independent University
14.25-14.45	Combinatorics of boundary singularities B_n^l of k-immersions and Bernoulli-Euler updown numbers	Oleg Karpenkov Moscow Independent University
14.55-15.15	Dini continuous probability weights and exact dimension of invariant measures	Anders Öberg University of Umeå
15.20-15.40	Interval dynamics and a model for drilling wells	Georgi Chakvetadze Steklov Institute, Moscow
	coffee break	
16.10-16.30	Examples of dynamical systems in finite field extensions of the p-adic numbers	Robert Nyqvist Växjö University
16.35-16.55	Heisenberg group over a field of finite characteristic and its factor-representations	Konstantin Kokhas St.Petersburg State University
17.00-17.20	A classification problem of manifolds with torus actions and given orbit space	Natalia Dobrinskaya Moscow State University
17.30-17.50	On non-existence of an iterative morphism which defines an Arshon sequence	Sergey Kitaev, Chalmers UT and Göteborg University
17.55-18.15	Countable symbolic systems	Sergey Shapovalov Moscow State University
19.00	Conference Dinner	

Saturday, October 7, E2 auditorium

Time	Title	Lecturer
9.00-9.50	Ergodicity and conservativity: random walks, Brownian motion, geometric flows (II)	Vadim Kaimanovich
10.00-10.50	Trends in percolation theory (II)	Olle Häggström
	coffee break	
11.20-11.50	Combinatorics of totally positive varieties	Michael Shapiro KTH Stockholm
12.00-12.20	Ergodic theorems for free group actions	Alexander Bufetov Princeton University, USA
	lunch	
14.00-14.20	Several ways to construct a normal form in the free solvable group	Sergey Dobrynin St.Petersburg State University
14.25-14.45	Metric properties of representations of the affine group	Mikhail Gorbulskij St.Petersburg State University
14.50-15.10	On the asymptotic behaviour of solutions to an ODE arising in cosmology	Hans Ringström KTH Stockholm
15.15-15.35	Some new examples in analytic ergodic theory	Maria Saprykina KTH Stockholm
15.45-16.05	Intrinsically hyperbolic maps and some properties of partially hyperbolic systems	Anton Gorodetski Moscow Independent University

Flows on surfaces and related geometric questions

Dmitry Anosov
Steklov Institute, Moscow

Trends in percolation theory

Olle Häggström
Chalmers University of Technology and Göteborg University

Percolation theory deals with connectivity properties of random media. As a mathematical idealization, one usually takes an infinite graph (such as the $\mathbb{Z}^d$ lattice), removes edges (or vertices) at random, and asks about the possible existence of infinite connected components in the remaining subgraph. Spectacular progress was made in mathematical percolation theory in the 1980's. This was followed by a more silent period in the first half of the 1990's. In the last few years, things have started to look better again, and the subject has began to develop in a variety of new directions. In this talk, I intend to survey some of these developments. Topics that I plan to discuss include: conformal invariance, continuum percolation, dynamical percolation, entanglement and rigidity percolation, and percolation on nonamenable graphs.

List of literature for further reading.

1. "Classical" percolation theory.

Grimmett, G.R. (1989) *Percolation*, Springer.

Grimmett, G.R. (1999) *Percolation* (2nd edition), Springer.

2. Conformal invariance.

Grimmett, G.R. (2000) Conformal invariance, droplets and entanglement, to appear in *State of the Art in Statistics and Probability Theory. Festschrift for Willem R. van Zwet* (M. de Gunst, C. Klaassen, and A. van der Vaart, eds.), IMS Lecture Notes.
`http://www.statslab.cam.ac.uk/~grg/preprints.html`

Werner, W. (2000) Critical exponents, conformal invariance and planar Brownian motion, to appear in *Proceedings of the 3rd Europ. Congress of Mathematics*, Birkhäuser.
`http://www.math.u-psud.fr/~werner/pre.html`

3. Percolation beyond $\mathbb{Z}^d$.

Benjamini, I. and Schramm, O. (1996) Percolation beyond $\mathbb{Z}^d$, many questions and a few answers, *Electron. Comm. Probab.* **1**, 71-82.
`http://www.math.washington.edu/~ejpecp/EcpVol1/paper8.abs.html`

Häggström, O., Peres, Y. and Schonmann, R.H. (1999) Percolation on transitive graphs as a coalescent process: relentless merging followed by simultaneous uniqueness, in *Perplexing Probability Problems: Festschrift in Honor of Harry Kesten* (M. Bramson and R. Durrett, eds.), pp 69-90, Birkhüser.
`http://www.math.chalmers.se/~olleh/papers.html`

Benjamini, I., Lyons, R., Peres, Y. and Schramm, O. (1999) Critical percolation on any nonamenable group has no infinite clusters, *Ann. Probab.* **27**, 1347-1356.
`http://www.wisdom.weizmann.ac.il/~schramm/papers`

Lyons, R. and Schramm, O. (1999) Indistinguishability of percolation clusters, *Ann. Probab.* **27**, 1809-1836.
`http://www.wisdom.weizmann.ac.il/~schramm/papers`

Lyons, R. (2000) Phase transitions on nonamenable graphs. *J. Math. Phys.* **42**, 1099-1126.
`http://www.php.indiana.edu/~rdlyons/`

Pak, I. and Smirnova-Nagnibeda, T. (2000) On non-uniqueness of percolation on nonamenable Cayley graphs, *Comptes Rendus Acad. Sci. Paris, Ser. I Math*, **330**, 495-500.
`http://www-math.mit.edu/~pak/research.html`

4. Dynamical percolation.

Häggström, O., Peres, Y. and Steif, J. (1997) Dynamical percolation, *Annales de l'Inst. H. Poincaré, Probab. Statist.* **33**, 497-528.
http://www.ma.huji.ac.il/~papers/recent.html

Häggström, O. (1998) Dynamical percolation: early results and open problems, *Microsurveys in Discrete Probability* (D. Aldous and J. Propp, eds.) pp 59-74, AMS, Providence, RI.

5. Continuum percolation.

Meester, R. and Roy, R. (1996) *Continuum Percolation*, Cambridge University Press.

Benjamini, I. and Schramm, O. (1998) Exceptional planes of percolation, *Probab. Th. Relat. Fields* **111**, 551-564.
http://www.wisdom.weizmann.ac.il/~schramm/papers

Jonasson, J. (2000) Optimization of shape in continuum percolation, to appear in *Ann. Probab.*
http://www.math.chalmers.se/~jonasson/recent.html

Menshikov, M., Rybnikov, K. and Volkov, S. (2000) Loss of tension in an infinite membrane with holes distributed by Poisson law, preprint.
http://www.fields.utoronto.ca/cgi-bin/preprint?refno=FI-PIA1999-017

6. Entanglement percolation.

Grimmett, G.R. and Holroyd, A.E. (1999) Entanglement in percolation, to appear in *J. London Math. Soc.*
http://www.math.ucla.edu/~holroyd/research.html

Häggström, O. (1999) Uniqueness of the infinite entangled component in three-dimensional bond percolation, to appear in *Ann. Probab.*
http://www.math.chalmers.se/~olleh/papers.html

Brownian limits for combinatorial problems

Svante Janson
Uppsala University

Many problems on random (discrete) combinatorial structures have asymptotics that can be described using Brownian motion and processes derived from it, such as the Brownian bridge and Brownian excursion. I will discuss some such problems, involving random trees, forests and applications to hashing in computer science.

Random matrices, random growth and random tilings

Kurt Johansson
KTH Stockholm

Random matrices have a rich mathematical structure with connections to many different parts of mathematics. Recently, connections with certain problems in combinatorial probability have been the focus of much attention. In this minicourse I will give some background on random matrices and discuss how random matrix distributions and random matrix-like distributions appear in certain random growth models, related to the length of the longest increasing subsequence in a random permutation, and in some types of random tilings.

Ergodicity and conservativity: random walks, Brownian motion, geometric flows

Vadim Kaimanovich
Université de Rennes, France

Ergodicity and conservativity are the most basic and fundamental properties of a group of transformations (particular case: just a single invertible transformation) with a quasi-invariant measure. The aim of the minicourse is to discuss these properties in two closely related situations:

- for Markov processes on algebraic and geometric structures (random walks on groups, Brownian motion on Riemannian manifolds);
- for geometric flows on negatively curved manifolds (the geodesic and the horocycle flow).

In particular, it will be shown that ergodicity of the both geometric flows admits a natural interpretation in terms of the Brownian motion on the manifold (recurrence for the geodesic flow and absence of bounded harmonic functions for the horocycle flow).

The problem of reversibility and polymorphisms which are quasisimilar to automorphisms

Anatoly Vershik
Steklov Institute, St.Petersburg

A polymorphism is a "multivalued map" in the category of measure spaces; also this is an analogue of Markov map for such spaces. The dynamic theory of polymorphisms is a generalization of the classical ergodic theory with tight links to the stationary Markov processes. Ergodic theory of the polymorphisms includes

- Geometrical part: the set of polymorphisms is a convex semigroup compact in weak topology.
- Spectral theory and theory of dilations.
- Entropy theory.
- Connection to (Kolmogorov) Markov processes.
- Trajectory theory and connections to gruppoids and C*-algebras.

Some of those topics will be discussed in the lecture.

On invariant manifolds in singularly perturbed systems

Olga D. Anosova
Moscow State University

A singularly perturbed system with a small parameter ε, with the slow variable y and with the fast variable x is considered:

$$\begin{aligned} \dot{x} &= f(x, y, \varepsilon), \\ \dot{y} &= \varepsilon g(x, y, \varepsilon). \end{aligned}$$

The hypothesis is that for all y from some bounded domain D the fast subsystem has a manifold $M_0(y)$ — hyperbolic and invariant or stable invariant and overflowing, — and that the motions in this system going in the directions transversal to $M_0(y)$ are faster than the mutual approaching of trajectories on $M_0(y)$. In the first case hyperbolicity of the invariant under flow g^t in R^n manifold M_0 means that the restriction of the tangent $TR^n|M_0$ splits into the Whitney sum of dg^t-invariant vector splittings $TM_0 \oplus E^{uu} \oplus E^{ss}$ such that

1) $dg^{-t}(a)\eta \to 0$ as $t \to \infty$ for all $\eta \in E_a^{uu}$,
2) $dg^{t}(a)\eta \to 0$ as $t \to \infty$ for all $\eta \in E_a^{ss}$.

To describe the velocity of the motions we introduce appropriate Lyapounov-type characteristic numbers:

$$\lambda_N^{(s)} = \inf_{a \in M_0} \varliminf_{t \to +\infty} \frac{-\log \|dg^t(g^{-t}a)|E_a^{ss}\|}{t},$$

$$\lambda_T^{(s)} = \sup_{a \in M_0,\, \xi \in T_a M_0} \varlimsup_{t \to +\infty} \frac{\log |dg^{-t}(a)\,\xi|}{t}, \qquad \xi \neq 0.$$

For E^{uu} and TM_0 time reversal we introduce analogous characteristic numbers $\lambda_N^{(u)}, \lambda_T^{(u)}$.

It is proved that if there exist the positive integer r such that $r\lambda_T^{(s)} < \lambda_N^{(s)}$ and $r\lambda_T^{(u)} < \lambda_N^{(u)}$, then for sufficiently small ε the full system has an invariant manifold close to $\bigcup_{y \in D} M_0(y) \times \{y\}$; the degree of its smoothness is specified.

Precise statement for overflowing stable invariant M_0 is given in the article with the same title in J. Dyn. Contr. Systems, vol.5, num.4, 1999

Particular cases of this theorem were proved by N. Fenichel (in the case when $M_0(y)$ is a dot) and K. Alymkulov (in the case when $M_0(y)$ is a closed orbit). However, in these cases important extra information is available: one can derive approximate differential equations of lower order describing the evolution of y. (In these cases, using the corresponding invariant manifold, one reduces the problem on singular perturbations to the problem on regular perturbations or on averaging.) Perhaps in other cases one would need some serious additional requirements on the fast system in order to do this. Right now we can say that in case of stable invariance the number of essential variables decreases by codimension of $M_0(y)$ in x-space.

On a relation between time average and minimal attractors

Pavel Bachurin
Moscow State University

Consider a compact of a positive Lebesgue measure in Euclidian space and a diffeomorphism that maps this compact strictly into itself. For such systems there is a notion of minimal attractor, introduced by Gorodetski and Ilyashenko. We prove that weak time averages of a continuous function on the compact depend only on the restriction of this function to the minimal attractor.

Discrete quasi-periodic Schrödinger equations

Kristian Bjerklöv
KTH Stockholm

We give an idea of how to prove that the one-dimensional quasi-periodic Schrödinger equation

$$-(u_{n+1} + u_{n-1}) + K \cdot E(\theta + n\alpha) u_n = 0$$

with a piecewise analytic potential E has a positive Lyapunov exponent for large values of the constant K and α in a set of positive Lebesgue measure.

Global bifurcations on the Klein bottle. Unimodal case

Anton R. Borisyuk
Moscow State University

In this article we consider bifurcations of compact homoclinic surfaces of nonhyperbolic cycle with property that the only multiplier equals one. Suppose X_ε is a one parameter family of C^∞ vector fields in R^4. A vector field appropriated to the critical parameter value X_0 has a nonhyperbolic cycle L, and one multiplier of the cycle equals one, and all other multipliers are outside of an unit circle. Such cycle is called a saddlenode cycle. We consider the case when the Klein bottle itself is the union of the cycle L and its homoclinic orbits. The problem is to find a full bifurcation scenario for the family of vector fields on the Klein bottle after disappearing of the saddlenode cycle.

To solve this problem we consider a *global Poincare map* of the Klein bottle having the following form:

$$\Delta_\varepsilon = \Delta_\varepsilon^{sing} \circ \Delta_\varepsilon^{reg} = -x + T(\varepsilon) + h(x;\varepsilon). \tag{1}$$

$T(\varepsilon)$ tends to infinity and the family of functions $h(x;\varepsilon)$ converges to some function $h_0(x)$ in norm of the space C^3 if parameter value tends to zero.

Here we consider the unimodal function h_0 (hence, the functions $h(x;\varepsilon)$ are also unimodal for small values of parameter ε). The function h is said to be unimodal if h is Morse function with one maximum and one minimum for period.

If we consider a new parameter $a = T(\varepsilon)$, then the global Poincare map is described by the following formula:

$$\tilde{f}_a:\ x \to\ -x + a + h_a(x), \tag{2}$$

where $x \in S^1,\ h_a(x) = h(x;\varepsilon)$. The map (2) is a diffeomorphism if and only if

$$h_a'(x) < 1. \tag{3}$$

The main results of the paper are:

1. We find a complete description of the bifurcation scenario for the family of diffeomorphisms (2)-(3) if the function h_a does not depend on parameter.

2. We describe and classify all possible bifurcation scenarios for the system (2)-(3).

3. We show that all results are also correct if the function h_a does depend on parameter a and some constraints are satisfied.

It has been proved that an orientation reversing diffeomorphism of the circle has exactly two fixed points and does not have cycles of period greater than two [1].

To describe the bifurcation scenarios we find all pairs (a, x), where x is a point of 2-cycle of the family $f_a : x \to -x + a + h(x)$ with parameter value a. We show that these and only these points belong to the graph of *cycle function* $a = p(x) = x + i(x) - h(x)$.

Definition 1. The dependence $i(x)$ is called an involution of the segment $[0; 2\pi]$ generated by unimodal function $h(x)$, if the following equation is correct for any $x : h(i(x)) = h(x)$.

Theorem 1. In the generic family $f_a(x) = -x + a + h(x)$ with unimodal function h the following bifurcations may occur.

1) Two orbits of period 2 appear if the parameter a crosses a local minimum of the cycle function p which is not coincide with a critical point of the function h. One of these orbits is stable and another one is unstable.

2) Two orbits of period 2 merge and disappear if the parameter a crosses a local maximum of the cycle function p which is not coincide with a critical point of the function h. One of these merging orbits is stable and another one is unstable.

3) A period doubling bifurcation occurs if the parameter a crosses a local minimum of the cycle function p which coincides with a critical point of the function h. The fixed point of the map f_a loses its stability and a period 2 orbit appears.

4) A reverse period doubling bifurcation occurs if the parameter a crosses a local maximum of the cycle function p which coincides with a critical point of the function h. Herewith, the stable period 2 orbit disappears.

Lemma 1. Suppose $p(x)$ is the cycle function of the family $f_a(x)$ and x_1 is maximum of the function h. Then the function $p(x)$ satisfies the following conditions $(*)$:

1) $p(0) = 2\pi$; $x_1 < p(x_1) < 2x_1$;
2) $p'(0) = p'(x_1) = 0$; $p'(x) < 1$ for $x \in (0; x_1)$;
3) Let $p'_-(x) = \begin{cases} p'(x) & , \quad p'(x) < 0 \\ 0 & , \quad p'(x) \geq 0 \end{cases}$; $x \in [0; x_1]$

$A_- = |\int_0^{x_1} p'_-(x)dx|$. Then $A_- \leq 2\pi - x_1$.

Theorem 2. Any function $p(x)$, $x \in [0; x_1]$, satisfying the conditions $(*)$ on the segment $[0; x_1]$ can be continued on the segment $[0; 2\pi]$ as a cycle function of some family f_a.

Definition 2. Two bifurcation scenarios for the family $F_a(x) = -x + a + f(x)$ and the family $G_a(x) = -x + a + g(x)$ (here the functions f and g are unimodal) are called equivalent if their cycle functions are RL equivalent regarding to a homeomorphism of circle.

Definition 3. Suppose the function f is a Morse function, $x^1 < x^2 < \cdots < x^n$ are critical points of this function, and the values $f(x^1) \ldots f(x^n)$ are ordered by increase: $y_1 < \cdots < y_n$. A substitution σ_n is called a characteristic substitution of Morse function f if $\sigma_n(i) = j,\ y_j = f(x^i)$.

Theorem 3. Consider two families of diffeomorphisms of the circle: $F_a(x) = -x + a + f(x)$ and $G_a(x) = -x + a + g(x)$. The functions p_f and p_g are cycle functions of families F_a and G_a respectively, and σ_n^f and σ_n^g are characteristic substitutions of these functions. Bifurcation scenarios for families F_a and G_a are equivalent if and only if σ_n^f and σ_n^g are equal.

The statement of Theorem 3 is a corollary of Arnold's results [2].

[1] Ilyashenko, Li Weigu, Nonlocal Bifurcations, Math. Surveys and Monographs, vol. 66, AMS 1999.

[2] Arnold V. I. Bernoulli-Euler updown numbers associated with function singularities, their combinatorics and arithmetics, Duke Mathematical Journal. 1991. Vol.63. No.2. P.537-555.

Acknowledgment. I am grateful to my supervisor Prof. Yu.S.Ilyashenko for his help.

Ergodic theorems for free group actions

Alexander Bufetov
Princeton University, USA

New ergodic theorems are proved for measure-preserving actions of free semigroups and groups. They are derived from ergodic theorems for Markov operators. This approach allows to generalize the theorems of Grigorchuk, Nevo, and Nevo and Stein. The theorems obtained are then applied to some classes of markov groups, including free products and Vershik's locally free groups.

Interval dynamics and a model for drilling wells

Georgi Chakvetadze
Steklov Institute, Moscow

The object of study is a family of piecewise monotonic interval maps serving as a model for the process of rock destruction by drilling bit. These maps were introduced by A. Lasota and P. Rusek and depend on two parameters — a scalar and a function. We prove that for a "large" parameter set these maps belong to the class of piecewise smooth dissipative dynamical systems having a unique measure theoretic attractor with strong stochastic properties.

First we are looking for the (metric) chaos in the system and derive the conditions (on parameters) sufficient for the existence of an absolutely continuous invariant probability (acip) of the map under consideration. These conditions are "almost necessary" as functional parameter is C^2–close to quadratic one. Once having obtained the acip we restrict our attention to its ergodic properties. We prove that the system is ergodic and weak mixing and, using well known results, we assert that it is exact, Bernoulli and satisfies the Central Limit Theorem for a broad class of functions.

The questions related to the stability of dynamics in the system under small random perturbations are also discussed.

[Ch1] Chakvetadze G., J. Dyn. and Cont. Sys., Vol. 2 (1996), 485-502.

[Ch2] Chakvetadze G., J. Dyn. and Cont. Sys., Vol 6 (2000), 75-95.

[ChS] Chakvetadze G., Stepin A., Int. J. of Bif. and Chaos, Vol 9 (1999), 1705-1718.

A classification problem of manifolds with torus action and given orbit space

Natalia Dobrinskaya
Moscow State University

Let M^{2n} be a smooth oriented manifold with smooth action of n-dimensional compact torus T^n. It's called *quasitoric* if the action is locally isomorphic to the standard one of T^n on C^n and the orbit space is diffeomorphic as manifold with corners to a simple n-dimensional polytope. The class of quasitoric manifolds contains all algebraic nonsingular projective toric varieties, but is much wider.

We find the necessary and sufficient conditions for a quasitoric manifold over the product of several simplices to be the space of an equivariant bundle with quasitoric fiber and base. The conditions are formulated in combinatorial terms and use the construction of a "coloring" of polytope's vertices. This criterion doesn't hold true for an arbitrary polytope (we construct a counterexample in dimension 3).

This result provides a classification of quasitoric manifolds over the product of simplices with positive coloring (note that all compact nonsingular toric varieties in algebraic geometry admit such a coloring).

Several ways to construct normal form in the free solvable group

Sergey Dobrynin
St.Petersburg State University

We construct a natural, but non-stabilized normal form in the free solvable group. To minimize this form and make it stable we change our construction using geometrical interpretation of the group. We prove several structural theorems in terms of sets of edges of the lattice in this geometrical description.

Building the tree of life

Niklas Eriksen
KTH Stockholm

With the HUGO project, biologists are given access to an enormous amount of data. In the interpretation of this data combinatorics can help. In this talk I will discuss how the theory of permutations can contribute to reconstruction of the tree of life.

Topological entropy for random dynamical systems and the analog of Misiurewicz-Przytycki theorem

Alexei Yu. Fishkin
Moscow State University

The subject of this work is topological entropy for the action of several continuous transformations of a metric compact. This notion of entropy was introduced by A. Bufetov[1]. In the situation of smooth orientable manifold and C^1 - smooth transformations we give the lower bound for the value of entropy.

Denote by W_m the set of all finite words in the alphabet $\{1, 2, \dots, m\}$. For the word $w = w_1 w_2 \dots w_n \in W_m$ denote by $|w|$ its length n and denote by $w^{(k)} = w_1 w_2 \dots w_k$ its head of length $k \leq n$.

Let X be a metric compact, let d be a distance function on X and let $f_1, f_2, \dots, f_m$ be continuous transformations of X. These mappings generate the semigroup $\langle f_1, f_2, \dots, f_n \rangle$ with respect to composition. If $w \in W_m$ and $|w| = n$, than we put $f_w = f_{w_n} \circ \dots \circ f_{w_1}$ and $f_\lambda = id_X$ for the empty word λ. For every word w we can define the distance function on X by the formula:

$$d_w(x_1, x_2) = \max_{0 \leq k < |w|} d(f_{w^{(k)}}(x_1), f_{w^{(k)}}(x_2)).$$

Definition. *Subset S of X is called ε - separated w.r.t. distance function d if for any two points $x, y \in S$ $d(x, y) > \varepsilon$.*

Denote by $N(w, \varepsilon)$ the maximum number of points in an ε - separated subset of X w.r.t. the distance d_w and put

$$N(n, \varepsilon) = \frac{1}{m^n} \sum_{w \in W_m, |w| = n} N(w, \varepsilon).$$

Definition[1]**.** *Topological entropy of the semigroup $\langle f_1, f_2, \dots, f_m \rangle$ is given by formula:*

[1] Probability theory and its applications, 3, pp. 696-697, 1999.

$$h(f_1, \dots , f_m) = \lim_{\varepsilon \to 0} \limsup_{n \to \infty} \frac{\log N(n, \varepsilon)}{n}.$$

The following theorem generalizes the theorem of Misiurewicz-Przytycki for the situation of the semi-group action.

Theorem. *Let X be a smooth compact orientable manifold and let $f_i : X \to X$, $i = 1, \dots , m$ be C^1 - smooth mappings. Then*

$$h(f_1, \dots , f_m) \geq \log \frac{|deg f_1| + |deg f_2| + \dots + |deg f_m|}{m}.$$

Remark. In the situation when all mappings increase distances between points of X the equality holds.

New applications of Bohnenblust-Spitzer combinatorial identity: joint distributions for random walks and a three-term Szego formula

Dimitri Gioev
KTH Stockholm

Let $S_n = X_1 + \dots + X_n$ be the position of a random walk with independent identically distributed steps at an integer time n, and set $T_n = \max(0, S_1, \cdots , S_n)$.

We obtain an expression for the bivariate characteristic function of S_n and T_{n+p}, $p \geq 1$, and compute the third asymptotic term in Szego's formula for a sphere in any dimension. Our main tool is an equivalent form of Bohnenblust-Spitzer combinatorial identity, for which we have an independent proof. Another application of our formula is an explicit expression for the moment of T_n of any integer order $m \geq 1$, which involves the Bell polynomials coefficients. We also obtain estimates for the covariance and correlation coefficient of S_n and T_{n+p}, as $n, p \to \infty$.

The equivalence of notions of entropy for the case of diadic sequences of partitions

Alexander Gorbulskij
St.Petersburg State University

The notion of entropy introduced by Vershik happens to coincide with its slight modification, which uses the notion of epsilon-entropy. This holds because of the special scaling used in that definition. Changing this scaling we obtain some more sensitive definitions that degenerate sometimes.

The metric properties of the representation of affine group

Mikhail Gorbulskij
St.Petersburg State University

In Kolmogorov's work different embeddings of $\mathbb{R}$ into a Hilbert space are considered. These embeddings can be obtained as degenerated orbits of representations of affine group. Different representations give isometric embeddings of $\mathbb{R}$ with special metrics.

Intrinsically hyperbolic maps and some properties of partially hyperbolic systems

Anton Gorodetski
Moscow Independent University

To study the properties of partially hyperbolic maps the notion of intrinsically hyperbolic map is useful. As "an Axiom $A^{\#}$" it was introduced by Alexeev and Yakobson to study Markov partitions and symbolic dynamics for homeomorphism of a metric space. Theorem 1 gives one more properties of intrinsically hyperbolic maps. It can be applied to the theory of partially hyperbolic sets because in some cases a map induced on a space of central leaves is intrinsically hyperbolic.

Let (Λ, d) be a compact metric space, $f : \Lambda \to \Lambda$ be a lipschitzemorphism (i.e. f is a homeomorphism, f and f^{-1} are Lipschitz continuous).

Let us define two equivalence relations $\sim_u$ and $\sim_s$ on Λ in the following way:

$$x \sim_u y \Longleftrightarrow d(f^{-n}(x), f^{-n}(y)) \xrightarrow[n\to+\infty]{} 0,$$

$$x \sim_s y \Longleftrightarrow d(f^{n}(x), f^{n}(y)) \xrightarrow[n\to+\infty]{} 0.$$

Let us also define local stable and unstable manifolds:

$$W^s_\varepsilon(x) = \{z | z \sim_s x \quad and \quad d(f^n(x), f^n(y)) \le \varepsilon \quad for \quad any \quad n \in \mathbb{Z}^+\},$$

$$W^u_\varepsilon(x) = \{z | z \sim_u x \quad and \quad d(f^{-n}(x), f^{-n}(y)) \le \varepsilon \quad for \quad any \quad n \in \mathbb{Z}^+\}.$$

Definition. A lipschitzemorphism f of a compact metric space (Λ, d) is *intrinsically hyperbolic* iff there exist $\varepsilon > 0, \delta > 0$ such that the following hold:

(IH1) If $d(x,y) < \delta$ then there exists a unique $w = W^u_\varepsilon(x) \cap W^s_\varepsilon(y) \in \Lambda$. The map $(x,y) \mapsto w(x,y)$ is continuous.

(IH2) There exist constants $C > 0,\ 0 < \nu < 1,\ 0 < \xi < 1$ such that

$$y \in W^s_\varepsilon(x) \implies d(f^n(x), f^n(y)) \le C\nu^n d(x,y), \quad n \in \mathbb{N},$$

$$y \in W^u_\varepsilon(x) \implies d(f^{-n}(x), f^{-n}(y)) \le C\xi^n d(x,y), \quad n \in \mathbb{N}.$$

The simplest examples are provided by usual locally maximal hyperbolic sets and the Bernoulli shifts.

Theorem 1. *Let Λ and Λ' be compact metric spaces, lipschitzemorphisms $f : \Lambda \to \Lambda$ and $f' : \Lambda' \to \Lambda'$ are intrinsically hyperbolic and $h : \Lambda \to \Lambda'$ a topological conjugacy: $h = f'hf^{-1}$. Then both h and h^{-1} are Hölder continuous.*

Now let B be the locally maximal hyperbolic set of the map $F : N \to N$, $B = \cap_{n\in\mathbb{Z}} F^n(U(B))$. Let M be a closed manifold.

Theorem 2. *Let the map $\mathfrak{F} : \mathfrak{N} \times \mathfrak{M} \to \mathfrak{N} \times \mathfrak{M}$, $(\mathfrak{x}, \varphi) \mapsto (\mathfrak{B}(\mathfrak{x}), \varphi)$ be of class C^r, $1 \le r < \infty$. Then any C^{r+Lip}-diffeomorphism $\mathfrak{G}$, which is C^r-close to $\mathfrak{F}$, has an invariant subset Δ, homeomorphic to $B \times M$; the projection $\Phi : (\Delta, \mathfrak{G}) \to (\mathfrak{B}, \mathfrak{F})$ is a semiconjugacy, the leaves $\Phi^{-1}(x)$ are C^r-smooth and depend Hölder continuously on the point $x \in B$ in C^r-norm.*

The idea of the proof is to consider the set of central leaves as a metric space. Any leaf is the point of this space, the metric is given by the C^r-distance between leaves. It appears that the induced map on this space is intrinsically hyperbolic. So Theorem 1 gives Theorem 2 in one step.

Theorem 2 also has interesting applications ([1], [2]).

[1] A. Gorodetski, Yu. Ilyashenko, Some new robust properties of invariant sets and attractors of dynamical systems, Functional Anal. and Appl. Vol. **33** (1999), 16-30.

[2] A. Gorodetski, Yu. Ilyashenko, Some properties of a skew products over horseshoe and solenoid, Proc. Steklov Inst. Math. (to appear)

A generalization of the law of large numbers for random matrices

Stefan Israelsson
Uppsala University

The aim of this talk is to shed light on the following problem: Is there an a such that

$$\lim_{n\to\infty} P(| \frac{1}{n} tr(A^n f(X^n)) - a | > \epsilon) = 0,$$

where

A^n	is a nonrandom matrix,
X^n	is a random symmetric matrix,
f	is a function.

This problem is solved for the case when we may write X^n as the following sum:

$$X^n = \Sigma^n + \sum_i^N B_i^n Y^n C_i^n,$$

where

$\varepsilon_{ij} = \varepsilon_{ji}$	are i.i.d with $\mathbb{E}(\varepsilon) = 0, Var(\varepsilon) = 1/\sqrt{n}$
$Y^n = (\varepsilon_{ij})$	is an $n \times n$ symmetric matrix (Wigner matrix)
B_i^n, C_i^n, Σ^n	are $n \times n$ matrices.

If f is bounded and smooth enough, A^n, B_i^n, C_j^n are uniformly bounded in the norm for all i, j; and if

$$\frac{1}{n} tr\left(B_i^n C_j^n \left(zI_n + \Sigma^n + \sum_{k,l} z_{kl} B_k^n C_l^n\right)^{-1}\right)$$

converge in probability for all i, j;

and $Z = (z_{ij})$ such that $Im(Z) > -Im(z)$;

then we can conclude the convergence of our limit to a certain number a.

Explicit isoperimetric constants for planar graphs and applications to percolation theory

Johan Jonasson
Chalmers University of Technology and Göteborg University

Let $G = (V, E)$ be an infinite locally finite connected graph. The (or rather one) edge-isoperimetric constant of G is defined as $\kappa(G) = \liminf_{N\to\infty}\{|\partial_E W|/|W| : W \subset V, N < |W| < \infty, W \text{ connected}\}$, where $\partial_E W$ is the outer edge-boundary of W. We say that G is amenable if $\kappa(G) = 0$ and in case $\kappa(G) > 0$ we say that G is nonamenable.

In percolation theory there are fundamental differences between amenable and nonamenable graphs. In many situations the particular value of $\kappa(G)$ for a nonamenable graph G is not very interesting, but in some situations it is of crucial importance to know (or being able to estimate) $\kappa(G)$. I will give examples of such situations and show how to compute $\kappa(G)$ exactly for planar biregular graphs.

Combinatorics of boundary singularities B_n^l of k-immersions and Bernoulli-Euler updown numbers

Oleg Karpenkov
Moscow State University and Moscow Independent University

Consider an M-region — a subset of polynomials with all real critical points in the space of real polynomials $x^{n+1} + a_{n-1}x^{n-1} + \ldots + a_1 x$. Stratum (a set of polynomials with multiple critical values) and caustic (a set of polynomials with multiple critical points) divide the M-region into new regions. It is sometimes called polyhedron of the snakes.

It was proved by V. I. Arnold that number of the regions, i.e., of the faces of zero codimension, corresponds to the M-region polynomials in general position is K_n, where

$$\sum_{n=0}^{\infty} K_n \frac{t^n}{n!} = \sec t + \tan t.$$

Using this result the author calculates numbers of polyhedron faces of some low codimensions corresponding to M-morsifications of B_n^l boundary singularities of k-immersions. M-morsification is some polynomial with special degenerations.

Result is given in K_i terms.

On non-existence of an iterative morphism which defines the Arshon Sequence

Sergey Kitaev
Chalmers University of Technology and Göteborg University

In 1937, Arshon gave us the construction of a symbolic sequence w, in the alphabet $\langle 1, 2, 3\rangle$, which is built as follows:

$w_1 = 1$; and w_{k+1} is attained from w_k by replacing
the letters in the odd positions as $1 \to 123$, $2 \to 231$, $3 \to 312$;
and the letters in the even positions as $1 \to 321$, $2 \to 132$, $3 \to 213$.

Then $w_2 = 123$, $w_3 = 123\ 132\ 312$, and each w_i is an initial subword of w_{i+1}, so the infinite symbolic sequence $w = \lim_{n\to\infty} w_n$ is defined. It is called the Arshon sequence.

The Arshon paper [1] was published in connection with the problem of building a non-repeated sequence in the 3-letter alphabet, that is a sequence which does not contain any subwords of the type $XX = X^2$, where X is any word in this alphabet. The sequence w has that property. The question of the existence of such a sequence was studied, in particular, in dynamical systems.

Let Σ be an alphabet. A map $\varphi : \Sigma^* \to \Sigma^*$ is called a morphism, if we have $\varphi(uv) = \varphi(u)\varphi(v)$ for any $u, v \in \Sigma^*$.

Study is now made of classes of sequences, which are defined by iterative schemes, and there are many techniques to study sequences generated by morphisms. Since the construction of the Arshon sequence w is similar to the iteration morphism scheme, and because w is constructed by two morphism f_1 and f_2, applied depending on whether the letter position is even or odd, we might expect that there exists a morphism f which generates w.

It turns our that the following theorem holds.

Theorem. *There does not exist a morphism, which generates the Arshon sequence.*

[1] Arshon S. E., The Proof of the Existence of n - letter Non-Repeated Asymmetric Sequences, Math. collected papers, New series, AS USSR, **3** no. 3 (1937), 769-779.

[2] Salomaa A., Gems of the Theory of Formal Languages, Mir, Moscow, 1986

[3] Klepinin A., Suhanov E., On combinatorial properties of Arshon Language, Discrete Analysis and Research of Operations, (1999)

[4] Lotaire M., Combinatorics on Words, Encyclopedia of Mathematics and its Applications Reading, MIT: Addison-Wisleu Publ. Co., (1986)

[5] Handbook of Combinatorics

On uniqueness of solution of Riemann-Hilbert problem: analytical classification of Fuchsian singular points

Victor Kleptsyn
Moscow State University

The result of joint work of author with B. Rabinovich will be described: the problem of analytical classification of Fuchsian singular points will be reduced to algebraic problem of investigating orbits of uppertriangular group actions on niltriangular matrices.

Heisenberg group over a field of finite characteristic and its factor-representations

Konstantin Kokhas
St.Petersburg State University

We describe the Grothendieck group, traces and factor-representations of Heisenberg group of 3×3 matrices with entries from a field with finite characteristic. To obtain this description we realize this group as inductive limit of increasing chain of finite groups and apply Bratteli diagrams technique. In this case Bratteli diagram is a graph a bit more complicated than a tree. Investigation of this graph allows us to construct all complex factor-representations in the same manner as representations of the classical Heisenberg group. An analog of Plancherel formulae for integration over group of additive characters with Haar measure is also valid.

An isotopic classification of spatial 3-valent graphs

Vitali Kurlin
Moscow State University

A finite graph is called **3-valent** if there are exactly 3 edges at each vertex. We consider only 3-valent non-oriented graphs, in general, non-connected with multi-edges and loops. A **spatial** graph is an embedding of a graph into $\mathbb{R}^3$ up to an ambient isotopy. Usually links and spatial graphs are represented by plane diagrams up to generalized Reidemeister's moves. Recently Dynnikov introduced a brilliant way to represent non-oriented links by 3-page diagrams. We extend his method to spatial 3-valent graphs. In order to formulate the main result consider the semigroup Stg with 24 generators $\mathbb{A} = \{a_i, b_i, c_i, d_i, e_i, f_i, g_i, h_i, i \in \mathbb{Z}_3\}$ and 90 relations ($i = 1, 2, 3$, we may throw out one of (4) by (3)) :

(1) $a_i = a_{i+1}d_{i-1}, \quad b_i = a_{i-1}c_{i+1}, \quad c_i = b_{i-1}c_{i+1}, \quad d_i = a_{i+1}c_{i-1},$
(2) $e_i = d_i g_i = g_{i-1}b_{i+1} = a_{i+1}h_{i-1}, \quad f_i = h_i b_i = d_{i+1}h_{i-1} = g_{i-1}c_{i+1},$
(3) $d_1 d_2 d_3 = 1,$
(4) $b_i d_i = d_i b_i = 1,$
(5) $(d_{i-1}c_{i-1})x = x(d_{i-1}c_{i-1})$, where $x \in \{c_i, e_i, b_{i-1}d_i d_{i-1}\}$,
(6) $uv = vu$, where $u \in \{a_{i-1}b_{i-1}, e_{i-1}b_{i-1}, b_{i+1}d_{i-1}d_{i+1}b_{i-1}\}, v \in \{a_i, b_i, c_i, e_i, b_{i-1}d_i d_{i-1}\}$.

Theorem. 1. Any spatial graph Γ is represented by some 3-page diagram, which corresponds to definite element $w_\Gamma \in Stg$.

2. Two 3-page diagrams Γ_1 and Γ_2 represent isotopic graphs, if and only if $w_{\Gamma_1} = w_{\Gamma_2}$ in Stg.

3. An element $w \in Stg$ is assigned to some spatial 3-valent graph, if and only if w is central, i.e. w commutes with each element of Stg.

In other words the center of Stg classifies all spatial 3-valent graphs up to an ambient isotopy. Let us show the sketch of proofs. We convert each plane diagram of a spatial 3-valent graph into a 3-page diagram (theorem 1). We encode each 3-page diagram by definite balanced word in alphabet $\mathbb{A}$. The sufficiency in theorem 2 follows from geometric realization of relations (1)-(6) by isotopy in $\mathbb{R}^3$. In order to prove necessity in theorem 2 we introduce some local moves of words in $\mathbb{A}$. We show that relations (1)-(6) generate these local moves and these moves generate any isotopy of graphs. Finally, we find the algebraic characterization of balanced words in $\mathbb{A}$ (theorem 3).

These results were first presented at the International Conference "Geometry and Applications", Novosibirsk (Russia), March, 2000. The author thanks V.M.Buchstaber for his support and I.A.Dynnikov for useful discussions.

The random assignment problem

Svante Linusson
University of Linköping

Let A be an $n \times n$ matrix with random independent variables in each entry, all of which are exponentially distributed with mean 1. Let $A(n)$ be the smallest possible sum of n entries in A such that no two are in the same column or the same row, i.e. it is a minimal assignment of A. $A(n)$ is then a random variable and we let $f(n)$ be the expected value of $A(n)$. The physicist Parisi has conjectured that $f(n) = 1 + 1/4 + 1/9 + ... + 1/n^2$. Since the conjecture was formulated the problem has attracted a lot of attention from many different research groups in physics, combinatorics, probability and optimization.

In a recent work with Johan Wästlund, we have one of the most far reaching results so far. I will explain our method to compute $f(n)$ recursively that we have used to give a computer aided proof of the conjecture for n less than or equal to 7. I will also present a new much more general conjecture, which we have proved in certain instances.

Percolation diffusion

Torbjörn Lundh
Chalmers University of Technology and Göteborg University

Let a Brownian motion in the unit ball be absorbed if it hits a set generated by a radially symmetric Poisson point process. The point set is “fattened” by putting a ball with a constant hyperbolic radius on each point. When is the probability non zero that the Brownian motion hits the boundary of the unit ball? That is, manage to avoid all the Poisson balls and “percolate diffusively” all the way to the boundary. We will show that if the bounded Poisson intensity at a point z is $\nu(d(0, z))$, where $d(\cdot, \cdot)$ is the hyperbolic metric, then the Brownian motion percolates diffusively if and only if $\nu \in L^1$.

Orbits of Zariski dense semigroups of $SL(n, Z)$

Roman Muchnik
Yale University, USA

Consider a standard action of $SL(n, Z)$ on the n-torus. It is known that for a matrix A in $SL(n, Z)$, whose eigenvalues are not roots of unity, the orbit of almost every point is dense in the n-torus. We show a similar and even stronger result for large semigroups of $SL(n, Z)$; namely that for a semigroup S that is Zariski dense in $SL(n, Z)$, the orbit of every non-rational point is dense in the n-torus.

Schreier graphs of self-similar group actions on the Cantor set

Volodymyr Nekrashevich
Kiev University, Ukraine

The talk will be about self-similar (fractal) group actions on the Cantor space and on the boundary of a rooted tree. Schreier graphs of such actions have self-replication properties similar to the properties of the self-similar tilings (like Penrose tilings) and often have exotic spectra of discrete Laplassian. We will talk about properties of these graphs and show a general method of constructing them, which generalizes the Vershik-Bratelli systems.

Geometry and analysis of de Rham's curve

Pavel Nikitin
St.Petersburg State University

De Rham's curve is a result of applying to a point of the plane one or another of two compressing linear transformations accidentally, infinitely many times. For two particular transformations it was studied by de Rham. We study properties of the curve formed by an arbitrary pair of transformations.

Examples of dynamical systems in finite field extensions of the p-adic numbers

Robert Nyqvist
Växjö University

The behaviour of discrete monomial polynomial dynamical systems is well known in both the fields of p-adic numbers $\mathbb{Q}_p$ and complex p-adic numbers $\mathbb{C}_p$. We give some examples as a first attempt to describe the behaviour of discrete non-monomial polynomial dynamical systems in a finite field extension of $\mathbb{Q}_p$. Our purpose is to see patterns which can help us to find a general theory for these dynamical systems. The examples presented here show that the basins of attraction and the Siegel disks for the fixed points in these dynamical systems have a very rich structure which depends greatly on the prime number p.

For instance, the dynamical system $f(x) = x^3 + x^2 - x - 2$ has the fixed point -1 and $\pm\sqrt{2}$. If $\sqrt{2} \notin \mathbb{Q}_p$ then we study the dynamical system in the finite extension $\mathbb{Q}_p(\sqrt{2})$. We found that $\sqrt{2}$ is indifferent for all p except $p = 17$, when it is an attractor. Also, the structure of the basin of attraction for -1 is complex. If, for example, $p = 23$ then the basin of attraction is the union of 2 disjoint open balls, but when $p = 29$ the basin of attraction is the union of 265 disjoint open balls.

On smooth models of dynamical systems with discrete spectrum

Alexander Prikhod'ko
Moscow State University

Let us consider a measure preserving flow (T^t) on a smooth manifold M given by a smooth vector field and leaving invariant a smooth invariant measure μ. We are interested in spectral properties of the flow (T^t) especially the case when the flow has discrete (pure pointwise) spectrum, i. e. possesses a full system of eigenfunctions. It is well known that in this case all eigenvalues are simple and form a countable discrete subgroup of $\mathbb{R}$. Our main result is the following

Theorem 1. For any countable subgroup $\Lambda \subset \mathbb{R}$ such that $\dim_{\mathbb{Q}} \Lambda < \infty$ there exists a C^∞ flow on a compact Riemannian manifold M without fixed points having pure pointwise spectrum Λ. The flow leaves invariant the natural measure on M.

We also establish the analogous existence theorem for diffeomorphisms.

Theorem 2. If $\Lambda \subset \mathbb{R}$ is a countable subgroup of $\mathbb{R}/\mathbb{Z}$ having finite dimension over $\mathbb{Q}$ then there exists a measure preserving diffeomorphism of a compact Riemannian manifold with pure pointwise spectrum Λ.

These theorems generalize the results due to Anosov and Katok on smooth models of irrational rotations [1] of tori and give new examples of smooth rank one systems (see [2]).

[1] Anosov D. V., Katok A. B. New examples in smooth ergodic theory. Ergodic diffeomorphisms. *Trudy MMO*, 1970, v. 23, 3–36.

[2] Ferenczi S. Systems of finite rank. *Colloquium math.*, 1997, v. 73, no. 1, 35–65.

[3] Kornfeld I. P., Fomin S. V., Sinaĭ Ya. G. Ergodic theory. Moscow, Nauka, 1980.

On the asymptotic behaviour of solutions to an ODE arising in cosmology

Hans Ringström
KTH Stockholm

In the standard cosmological models of the universe one assumes it to be spatially homogeneous and isotropic, and Einstein's equations then reduce to an ODE for one unknown function. If one drops the condition of isotropy, one gets a system of ODE:s. We consider the asymptotic behaviour of solutions to one such system, which has been conjectured to be a model case for general inhomogeneous cosmological singularities.

Some new examples in analytic ergodic theory

Maria Saprykina
KTH Stockholm

We present a construction method that can be used to obtain examples of a non-linearizable Lebesgue measure preserving analytic diffeomorphism of the two-torus, that is uniquely ergodic with respect to this measure.

Moreover we show that uniquely ergodic diffeomorphisms are generic in a certain subclass of analytic diffeomorphisms, conjugate to a rotation of the form $F_\alpha : (r, \varphi) \mapsto (r, \varphi + \alpha (mod\ 1))$, $\alpha \in \mathbb{R}$.

Euler characteristic of the spaces of real meromorphic functions

Sergei Shadrin
Moscow State University and Moscow Independent University

(1) Consider an arcwise connected component H of the space of real meromorphic functions. One can build its geometric compactification N. Our goal is to compute the Euler characteristic of N and H.

(2) I'm going to show a recently found relation between Euler characteristic and multiplicity of Lyashko-Looijenga map's restriction. I'd like to state some rather mysterious numerical results, that follow from the foregoing relation.

(3) Also I'll formulate some unsolved problems in combinatorics of graphs. Solutions of these problems make possible to compute Euler characteristic for all components of the space of real meromorphic functions. The expected answer is a number depending on the topological characteristics of functions from H.

Combinatorics of totally positive varieties

Michael Shapiro
KTH Stockholm

We give a natural geometrical construction of a regular cellular complex (cfg. A. Björner, Posets, regular CW complexes and Bruhat order, Europ. J. Combin., **5** (1984), 7–16) and deduce some properties of Kazhdan-Lusztig polynomials.

Countable symbolic systems

Sergey Shapovalov
Moscow State University

The subject under review is possible topological behaviors of symbolic dynamical systems with countable phase spaces.

The classification problem for symbolic dynamical systems has a long history (Williams, Kim, Roush, Wagoner). Since in the general case it still stays unsolved, D.V. Anosov initialized attempts to obtain a complete classification in the simplest case of symbolic dynamical systems with countable phase spaces.

Let Ω_2 be the space of all 2-sided infinite sequences consisting of 2 symbols: 0 and 1. The topology on Ω_2 is the Tikhonov topology of componentwise convergence. Let T be the shift map on Ω_2: for $x = \{x_i\}_{i=-\infty}^{+\infty}$, $y = \{y_i\}_{i=-\infty}^{+\infty}$ $Tx = y$ if and only if $y_i = x_{i+1}$ for all i. We consider dynamical systems (M, T) which are restrictions of the cascade $(T^n; n \in \mathbb{Z})$ onto countable closed shift-invariant subsets M of the space Ω_2.

In a more general case, let G be a lattice and let Ω_2 be the space of all configurations over the alphabet $A = \{0, 1\}$ (a configuration is a map from G to A). The action T of the group G on Ω_2 is defined by the formula $(T_g x)(h) = x(g^{-1}h)$, $x \in \Omega_2$, $g, h \in G$. In this situation countable symbolic systems (M, T) are restrictions of the action T onto countable closed (in Tikhonov topology) T-invariant sets $M \subset \Omega_2$.

We say that two dynamical systems (M_1, T) and (M_2, T) are topologically conjugate, if there exists a homeomorphism $h : M_1 \to M_2$ which commutes with the action T. The major problem is to classify countable symbolic systems up to topological conjugacy.

A possible approach to this problem was suggested by A.M. Stepin. He introduced a new topological invariant of countable dynamical systems which was called "depth". This invariant was studied by S.A. Shapovalov who also proved a theorem describing the set of all possible values of the depth.

Besides that, applying some modifications of the techniques used for the depth research, S.A. Shapovalov also obtained a new solution of Birkhoff center depth problem. Recall that the problem of finding all ordinal numbers which can be center depths of some dynamical systems was formulated in 1926 by Birkhoff and solved in 1940-th by A.G. Mayer. But the new solution seems to be more simple and comprehensive than the one Mayer obtained.

Nevertheless, the general classification problem for countable symbolic systems stays unsolved. It is proved that the system of two topological invariants, depth and center depth, is not full. There exists an example of two non-conjugate dynamical systems having equal depths and equal center depths.

On the fine structure of stationary measures in IFS which contract-on-average

Nikita Sidorov
University of Science and Technology in Manchester, UK

Suppose $\{f_1, \ldots, f_m\}$ is a set of Lipschitz maps of $\mathbb{R}^d$. We form the iterated function system (IFS) by independently choosing the maps so that the map f_i is chosen with probability p_i. We assume that the IFS contracts on average and give an upper bound for the Hausdorff dimension of the stationary measure induced on $\mathbb{R}^d$ and as a corollary show that the measure will be singular if the modulus of the entropy $\sum_i p_i \log p_i$ is less than d times the modulus of the Lyapunov exponent of the system. Using a version of Shannon's Theorem for random walks on semigroups we improve this estimate and show that it is actually attainable for certain cases of affine mappings of $\mathbb{R}$. This work is joint with D. Broomhead and M. Nicol.

Graphs with large girth

Maxim Vsemirnov
Steklov Institute, St.Petersburg

Let $X_{n,k}$ be a k-regular graph with n vertices and girth$(X_{n,k})$ denote its girth, i.e., the length of its shortest cycle. It is known that girth$(X_{n,k}) \leq 2\log_{k-1} n$. Probabilistic method due to Bollobas proves noneffectively the existence of graphs $X_{n,k}$ with girth$(X_{n,k}) \geq \log_{k-1} n$. Lubotzky, Phillips and Sarnak gave an explicit construction with even better lower bound girth$(X_{n,k}) \geq \frac{4}{3}\log_{k-1} n$. In this talk an improvement of their construction will be given. Namely, we find a family of graphs with girth$(X_{n,k}) \geq \frac{12}{7}\log_{k-1} n$. As a key step of our method we develop the arithmetic of integer octonions. In contrast with the Lubotzky-Phillips-Sarnak construction it is still not known if our graphs are Ramanujan graphs.

Further possible improvements will be also discussed.

Limit theorems for partitions

Yuri Yakubovich
Steklov Institute, St.Petersburg

We consider fluctuations of a Young diagram near its limit shape and prove central limit theorem and local limit theorem for these fluctuations. The boundary of a normalized Young diagram is (in the limit) a non non-stationary Gaussian process.

Dini continuous probability weights
and exact dimension of invariant measures

Anders Öberg
University of Umeå

We prove exact-dimensionality and calculate the exact dimension for invariant measures associated with non-overlapping iterated function systems with probability weights which are Dini continuous on a logarithmic scale. The main point is that the intervals do not shrink in a uniform rate (some of them don't shrink at all) and the low regularity imposed on the weight functions.