UNIVERSITÉ DE GENÈVE
FACULTÉ DE PSYCHOLOGIE ET DES SCIENCES DE L'ÉDUCATION

International Commission on Mathematical Instruction

DiMaGe
Didactique des Mathématiques à Genève

Teaching and learning Algebra - An international symposium

Geneva Wednesday 7th June 2017

Uni Mail 40 bd du Pont d'Arve room MR070

All presentations will be either in English with French slides or in French with English slides

Participation is free but inscription on line is compulsory: here

9h30-10h45

From Algebra to Functions: promoting mathematical thinking in the classroom

Ferdinando Arzarello

Università degli Studi di Torino, Dipartimento di Matematica "G. Peano"

Former President of ICMI

10h45-12h00

Revisiting school algebra through visualization and sense making

Abraham Arcavi

Weizmann Institute of Science, Israel

Secretary general of ICMI

12h-14h00

Lunch

14h00-15h15

Learning about teaching Algebra through Lesson Study

Jill Adler

SARChI Mathematics Education Chair - University of the Witwatersrand (South Africa),

President of ICMI - Hans Freudenthal ICMI Award 2015

15h30-16h45

Conception collaborative de ressources
pour l'enseignement de l'algèbre élémentaire

Sylvie Coppé

Maitresse d'enseignement et de recherche, Equipe DiMaGe, FPSE, UniGe

17h00 -18h15

Le développement de la pensée algébrique chez les jeunes élèves :
une approche vygotskienne

Luis Radford

Université Laurentienne Sudbury, Ontario, Canada

Vice président de l'ICMI – Hans Freudenthal ICMI Award 2011

Abstracts

From Algebra to Functions: promoting mathematical thinking in the classroom

Ferdinando Arzarello

One of the most delicate issues in the teaching/learning of mathematics is ensuring that students acquire the mind-set for grasping the mathematical sense of the teaching situations they face: researchers like A. Schoenfeld speak of mathematical sense making ([1]). It is the exact opposite of the image that many people have of mathematics as a set of rules and algorithms to be learned by heart to answer questions sometimes far from any real sense. It is because of the daily activities and practices in the classroom that such a sense grows up: students develop, more or less consciously and more or less consistently, but inexorably, (a protocol of) rules to follow, for example to succeed or at least to "survive" the questions of the teachers. Specifically, this is the way they mature their own sense for algebraic activities and this may be very far from the right one: plenty of their mistakes exemplify this.

My lecture will illustrate how one can design suitable learning situations and pursue classroom practices, which can generate a genuine mathematical sense for mathematical formulas through the intertwining of problem solving with problem posing. The goal is to propose a figure of the teacher, who is not (any longer) a transmitter of rules, but a promoter of mathematical sense making for students. The proposed teaching-learning method is called Method of Varied Inquiry (MVI): it is based both on the Method of Variation ([2]) and on the Logic of Scientific Inquiry, which dates back to Galileo, and can also make use of ICT.

The development of MVI in the classroom can serve both as a tool that develops an adequate vision of mathematics, recognized and appreciated as a backdrop to address significant problems, and as an antidote for the purpose of preventing / disrupting a vision of mathematics reduced to a set of rules to be stored and applied.

During the presentation I will illustrate MVI with concrete examples of classroom activities: some of them possibly will be linked to the presentation of prof. Arcavi, so that the audience can face similar problems from different standpoints.

Revisiting school algebra through visualization and sense making

Abraham Arcavi

Algebra is considered a central subject to be studied in junior high and high schools in almost all educational systems around the world. The reasons for this centrality may vary but the consensus seems to be that time and effort should be devoted to its teaching and learning. There are some students who succeed in algebra and enjoy studying it as they seem to “understand the rules of the game”, to be able to eschew the most common calculation mistakes, to manipulate and play with complicated algebraic expressions and

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1 Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.), *Handbook for Research on Mathematics Teaching and Learning* (pp. 334-370). New York: MacMillan.

2 Marton, F., Runesson, U., & Tsui, A. (2003). The space for learning. In: F. Marton & A. Tsui (Eds.), *Classroom discourse and the space for learning*, pp. 3- 40. Mahwah, NJ: Lawernece Erlbaum Associates, Inc.

to enjoy the gratification provided by a correct result or by the checking and redoing when it was incorrect. Some may even see a point in studying and applying it in non-mathematical contexts. However, many more, perhaps a majority of, students around the world have difficulties in becoming competent and they fail to see why they should study algebra at all. They view algebra mostly as a set of meaningless procedures obeying arbitrary syntactic rules far removed from common sense and mostly useless in their daily lives.

Some of the curricular approaches and/or classroom practices may have contributed to create such negative perspectives. However, there are approaches designed purposefully to stress meaning and motivation in order to appeal to a wider population of students. In my presentation, I will describe such approaches, their rationale and the tools, which have the potential to enliven the teaching and learning of algebra. In particular, I will describe and exemplify ways of stimulating sense-making by using visualization and other means. I will also share data from student ways to solve algebraic tasks in creative ways.

Analysis of a lesson study cycle about Algebra

Jill Adler

Lesson Study as a context for professional learning is now widespread in mathematics education. In the Wits Maths Connect Secondary Project (WMCS), a project working with secondary mathematics teachers in several districts in one province in South Africa, we use an adapted version of Lesson Study where collaborative teams of teachers and project researchers plan, reflect, replan and reteach a lesson. To date our Lesson Study work has been on topics in Algebra and Functions. We have developed a framework to guide planning and reflection through careful attention to how particular concepts and/or processes can be illuminated for learning through (1) careful choice and sequencing of tasks and the examples and representations within these, and (2) careful attention to talk – to how words are used to build justifications in mathematics in ways that make sense to learners, and build their mathematical discourse. This latter is critical in our multilingual context. With respect to exemplification, and teachers find this particularly helpful, we pay attention to variation, what can vary and what needs to remain invariant to enable learning, and thus share some similarities with other proposed presentations (e.g. Arzarello). In this seminar, I will describe a lesson study cycle, and how researchers and teachers work together, with a guiding framework, to build their respective knowledge of and practice in learning and teaching mathematics.

Conception collaborative de ressources pour l'enseignement de l'algèbre élémentaire

Sylvie Coppé

Depuis plus de 10 ans, nous travaillons dans un groupe de recherche collaborative (composé d'enseignants et de chercheurs) à l'élaboration et à la diffusion de ressources pour les professeurs et les formateurs de mathématiques dans le cadre de l'enseignement de l'algèbre élémentaire en France (élèves de 11 à 15 ans). L'objectif est d'élaborer des activités ayant un potentiel didactique important pour motiver l'emploi des notions algébriques mais aussi de travailler de façon conjuguée les aspects

techniques, le sens et les justifications. Nous utilisons notamment les programmes de calcul avec une gestion de classe originale. Ces documents sont disponibles sur le site http://pegame.ens-lyon.fr/ .
Dans cette communication, nous montrerons comment ce travail qui s'appuie sur des recherches nombreuses sur les aspects épistémologiques et didactiques de l'algèbre élémentaire prend également en compte les pratiques ordinaires des professeurs afin de proposer des ressources viables, réalisables dans les classes et diffusables. Nous nous attacherons à montrer les effets de ce travail sur les pratiques des enseignants et nous pointerons les nouvelles questions qui se posent aux chercheurs.

Le développement de la pensée algébrique chez les jeunes élèves : une approche vygotskienne

Luis Radford

Dans cette présentation, je m'appuie sur le concept dialectique de Vygotsky de développement pour rendre compte de l'émergence et de la croissance de la pensée algébrique de jeunes élèves (7-8 ans). Inspiré par le matérialisme dialectique, je suggère que la pensée n'est pas quelque chose produit par un esprit isolé ou solipsiste ; la pensée n'est pas non plus quelque chose qui se passe uniquement dans la tête. Penser est plutôt une pratique sociale matérialisée dans le corps (par exemple, par le biais d'actions kinesthésiques, gestes, perception, visualisation), dans l'utilisation de signes (par exemple, des symboles mathématiques, graphiques, des mots écrits et parlés) et d'artefacts de différentes sortes (règles, calculatrices, etc.). Psychologiquement parlant, la pensée est un *système* : une unité dynamique sensuelle d'*unités* à la fois matérielles et idéelles. Ces unités comprennent, entre autres, les formes culturelles d'imagination sensuelle et les actions réelles avec les artefacts culturels, la tactilité, les gestes, le discours (intérieure et extérieure). En suivant les travaux tardifs de Vygotsky, je suggère que l'élaboration d'un système psychic se trouve sur la dialectique entre organisation, réorganisation et expansion de ses unités. Dans cet ordre d'idées, le développement de la pensée est un processus de réorganisation et un développement par le biais de contradictions dynamiques, qui dérivent de la façon dont l'individu produit des sens à partir de chacune des unités de la pensée. Mon étude empirique de l'évolution de la pensée algébrique des jeunes élèves s'appuie sur des recherches longitudinales dans lesquelles une cohorte d'élèves de 2e année (âgés de 7-8 ans) a été suivie pendant cinq ans. Je me concentre sur les unités suivantes : perception, gestes, discours, artefact et usage des symboles et j'examine comment une dialectique de sublimation et de réorganisation a lieu alors que les élèves s'engagent dans des activités d'enseignement / apprentissage spécifique en classe avec leurs enseignants.