Regional Disparities and Urban Economics Problems (document under perpetual construction)

Frédéric Robert-Nicoud (GSEM

This version: 15 February 2018

1. Monocentric city model with substitution in consumption and production

Consider a linear 'monocentric' city populated by N identical households. All commute to the CBD and earn the same income y. Commuting involves a disutility cost $\mathrm{e}^{-\tau x}$ only, where e is Euler's constant, $\tau > 0$ is a parameter negatively related to the quality of the commuting technology and $x \geq 0$ is the distance between the residence of the household and the CBD; the city expands in one direction only (Long Island?). Households consume two goods, c and h; the price of the consumption bundle c is the same across the city and normalized to 1. We are interested in determining the price of housing h as a function of x. We denote this price by $p(x)$. Household preferences over distance to the CBD, housing and c are defined by

$$U(h,c,x) = \left(\frac{h}{\mu}\right)^{\mu} \left(\frac{c}{1-\mu}\right)^{1-\mu} \mathrm{e}^{-\tau x},$$

where μ is the share of housing expenditure. The budget constraint of a household living in location x is $y = c + p(x)h$.

Housing is produced by competitive real estate developers who produce housing using land T, capital K, and construction labor L. Capital and labor are elastically supplied at prices $i > 0$ and $\lambda > 0$. Let $r(x)$ denote the unit price of land. Land is allocated to the highest bidder and commands an alternative price $\rho > 0$. Housing technology is described by the following production function:

$$H = \left(\frac{T}{\alpha}\right)^{\alpha} \left(\frac{K}{\beta}\right)^{\beta} \left(\frac{L}{1-\alpha-\beta}\right)^{1-\alpha-\beta}, \tag{1}$$

where $\alpha,\beta \in (0,1)$ are technology parameters.

1. Show that the consumption and location choices of households can be written as

$$\max_{x,h} U(x,h) = \left(\frac{h}{\mu}\right)^{\mu} \left(\frac{y - p(x)h}{1-\mu}\right)^{1-\mu} \mathrm{e}^{-\tau x}. \tag{2}$$

2. Show that the first order condition of the problem above for x yields

$$p'(x)h(x)\frac{\partial U}{\partial c} = \frac{\partial U}{\partial x} \quad \text{or, equivalently,} \quad (1-\mu)p'(x)h(x)\frac{1}{c(x)} = -\tau,$$

and interpret. This is known as the *Alonso-Muth-Mills* condition.

3. Show that the first order condition for h of the problem in (2) yields

$$p(x)h(x) = \mu y \tag{3}$$

and interpret.

4. Show that the indirect utility associated with (2) may be written as

$$V(p,x) = \frac{y}{p(x)^{\mu}}\mathrm{e}^{-\tau x}.$$

5. Show that households are indifferent as to where to reside if and only if $p(x)$ obeys

$$p(x) = \left(\frac{\mathrm{e}^{-\tau x} y}{\omega}\right)^{1/\mu}, \tag{4}$$

where ω is the common level of utility. This price is the maximum amount of money they are willing to pay to achieve utility ω and reside in location x, given their income y. For this reason, this is known as *the bid rent curve* in this literature.

6. Interpret this expression. In particular, how does the sensitivity of p with respect to x change as μ and τ vary, and why?

7. Turn to producers. Show that (1) yields the following marginal cost in H:

$$mc(i,r,\lambda) = r^{\alpha} i^{\beta} \lambda^{1-\alpha-\beta}$$

so that marginal cost pricing (perfect competition) yields the following *bid rent curve* for land from developers:

$$r(x) = \left(p(x) i^{-\beta} \lambda^{-1+\alpha+\beta}\right)^{1/\alpha}. \tag{5}$$

8. Let $\bar{x}$ denote the urban fringe. Then

$$r(\bar{x}) = \rho \quad \text{and} \quad p(\bar{x}) = \pi, \quad \text{where} \quad \pi \equiv \rho^{\alpha} i^{\beta} \lambda^{1-\alpha-\beta} \tag{6}$$

denotes the marginal cost and the unit price of housing at the fringe. Why does the equality $r(\bar{x}) = \rho$ hold? Why does the equality $p(\bar{x}) = \pi$ hold?

9. Show that (4), (5), and (6) jointly yield

$$r(x) = \rho \exp\left(\tau \frac{\bar{x} - x}{\alpha\mu}\right) \quad \text{and} \quad p(x) = \pi \exp\left(\tau \frac{\bar{x} - x}{\mu}\right). \tag{7}$$

10. Show that solving (7) for $p(0)$ gives an expression for real consumption ω as a function of $\bar{x}$:

$$\omega(\bar{x}) = y\pi^{-\mu} \mathrm{e}^{-\tau\bar{x}}. \tag{8}$$

Why does this expression for welfare hold *at any location* x in the city? [Hint: compute the welfare of the household living at the city fringe, $\bar{x}$.]

11. Let $k \equiv K/T$ denote physical density and $f \equiv H/T$ denote the 'floor area ratio.' Using the expression above and the production function (1), show that this implies the following relationships between density and distance to the CBD, given $\bar{x}$:

$$k(x) = \frac{\beta}{\alpha i} \exp\left(\tau \frac{\bar{x} - x}{\alpha\mu}\right) \quad \text{and} \quad f(x) = \frac{\rho}{\alpha\pi} \exp\left(\tau(1-\alpha) \frac{\bar{x} - x}{\alpha\mu}\right).$$

Interpret.

12. By the same token, use (3) to get an equilibrium relationship between the size of dwellings demanded by individual households and their location relative to the CBD:

$$h(x) = h(\bar{x}) \exp\left(\tau \frac{x - \bar{x}}{\mu}\right), \quad \text{where} \quad h(\bar{x}) = \frac{\mu}{\pi} y.$$

13. We can now solve for the land market equilibrium. By choice of units for land, the density of land supply is 1 so that the total amount of land used to supply housing services in the city is $\bar{x}$. Also, the number of square foot of housing per square meter ground floor is equal to $f(x)$ by definition; the total demand for housing in the city is thus $\int_0^{\bar{x}} f(x)\mathrm{d}x$. We can exploit a useful property of the Cobb-Douglas utility function in (2) whereby aggregate expenditure on housing, $\int_0^{\bar{x}} p(x) f(x)\mathrm{d}x$, is equal to a constant share μ of aggregate income, $N \times y$ (note: in this problem we are agnostic as to whether y includes land rents or not; for simplicity, we treat y as a parameter). Show that this equality implies

$$Ny = \frac{\rho}{\tau}\left[\mathrm{e}^{\tau\bar{x}/(\alpha\mu)} - 1\right] \quad \text{or, equivalently,} \quad \bar{x} = \frac{\alpha\mu}{\tau} \ln\left(1 + \frac{Ny\tau}{\rho}\right), \tag{9}$$

and interpret.

14. Use (8) and (9) to solve for ω and get

$$\omega = y\pi^{-\mu}\left(1+\frac{\tau N y}{\rho}\right)^{-\alpha\mu}$$

and interpret.

15. For the aficionados only: in the expression above, the effect of y on ω is seemingly ambiguous (the net effect is actually positive) because y has two distinct effects on ω; which are they?

16. Please sign the following partial derivatives:

$$\frac{\partial \bar{x}}{\partial y}, \quad \frac{\partial \bar{x}}{\partial \tau} \quad \text{and} \quad \frac{\partial \bar{x}}{\partial N}$$

and interpret.

17. In turn, show that $r_0 \equiv r(0)$ is increasing in N and y, and interpret.

18. In the 'closed city' version of the model, the population of the city is exogenously given at N; use (8) to show that ω is increasing in y and decreasing in N and τ, and interpret.

19. In the polar case, known as the 'open city' version of the monocentric city model, N is elastically supplied as long as utility in the city yields ω (ω is the outside option in the urban system). What is this condition known as? Use (8) to show that N is increasing in y and decreasing in τ and ω, and interpret.

20. In an intermediate case, assume that N increases with the utility enjoyed in the city at constant elasticity, so that

$$N = \bar{N}\left(\frac{a\omega}{a_*\omega_*}\right)^{\eta}, \qquad (10)$$

where $\bar{N} > 0$ is a parameter (you may normalize it to unity if you wish), $\eta > 0$ is the elasticity in question, and $a > 0$ captures some non-market local amenities such as the presence of desirable natural amenities (nice views, mild climate), public services (good schools, parks), or social environment (friendly atmosphere, low crime rates); $a_*\omega_*$ denotes the welfare potential migrants would obtain in alternative cities. Note that this case encompasses the cases in 18 (in which case $\eta = 0$) and in 19 (in which case $\eta \to \infty$). Draw (8) and (10) in the (N,ω)-space. How would you label the intersection of these curves? How do N, r_0, and ω vary when τ falls? and when y or a increase?

2. Spatial competition

Former exam question (Regional and Urban Economics, mid-terms 2010-2011 and 2011-2012).

Consider a segment of length l occupied by a continuum of workers of uniform density δ (in other words there are $L = l\delta$ workers on this segment). There are two firms, 1 and 2, located in 0 and l. Their profits are $\pi_j = (A - w_j)\, L_j - F$, $j = 1{,}2$, where L_j is the number of workers hired by firm j and F is a fixed cost. To work for a firm located at distance x from her location on the segment, a worker must pay a training cost μx^2 and her utility is given by $\omega = w - \mu x^2$.

1. Given w_1 and w_2, what is the location on the segment of the indifferent worker between the two firms?
2. Compute the workforce and the profits for both firms for any w_1 and w_2 (assuming that w_1 and w_2 are not too different to avoid corner solutions).
3. What is the profit maximizing wage for firm 1 given w_2? What is the profit maximizing wage for firm 2 given w_1? Exhibit the Nash equilibrium of this game. Interpret briefly.
4. How do wages and worker utility evolve when l and δ increase? Interpret briefly.
5. Consider now that l adjusts so that firms make zero profit. Interpret this assumption.
6. Compute l as a function of the other parameters of the model. How do wages and worker utility evolve when δ increases? Interpret briefly.
7. To what category does this model belong? Is the theoretical relationship between w and δ borne out in the data?

3. Commuting technology and city size

Former exam question (Regional and Urban Economics, mid-term 2010-2011).

Continuous innovations in transportation technologies are having dramatic effects on the shape and sizes of cities. For instance, O'Sullivan reports that Boston grew spectacularly following the introduction of the streetcar in the late 19th century.

1. What kind of growth are we talking about (population, land use...)?
2. How can you explain such patterns?

3. In which way(s) the consequences of the invention of trucking were fundamentally different from those of mass transit systems such as the streetcar and the subway?

4. How did the inventions of the elevator and steel shape modern cities?

4. Henderson's model

Former exam question (Regional and Urban Economics, mid-term 2010-2011, retake 2010-2011, and mid-term 2011-2012).

1. If agglomeration economies take place between firms *within* sectors, explain why we expect cities to specialize in equilibrium. Can Henderson's model (published in 1974 in the American Economic Review) account for the existence of diversified cities?

2. Consider a region with a workforce of 12 million. The urban utility (or net wage) curve reaches its maximum with 3 million workers and includes the following combinations:

Workers (mio)	1	2	3	4	6	8	9	10	11	12
Utility ($)	32	56	70	65	55	45	40	35	30	25

Initially, there is a single city with 12 million workers. Suppose the regional government establishes a new city with 1 million workers, leaving 11 million workers in the old city.

a. Assume that the number of cities remains at two. What happens next? What is the new equilibrium city size?

b. Suppose instead that the government establishes three new cities, each with 1 million workers, leaving 9 million in the old city. What happens next? Will the region reach the configuration of four cities, each with 3 million workers?.

c. Suppose now that your objective is to reach the optimum configuration and you establish three new cities. What is the minimum number of workers to be placed initially in each of the new cities?

5. Urban system

Former exam question (Regional and Urban Economics, retake 2010-2011, Public Economics 2017).

Consider an economy endowed with a large number of sites suitable for urban development ('large' in the sense that many sites will remain empty in equilibrium) and indexed by $j \in \{1,...,J\}$ and a large number of individuals/workers ('large' in the sense that many sites will be populated in equilibrium). There are two primary factors in this economy, labor, denoted by L, and land, denoted by T. Individuals (workers) consume a single, homogeneous and perfectly tradable consumption good denoted by Y and a single non-tradable service, housing, denoted by H. Preferences of the representative consumer are summarized by the following utility function: $U(h,y) = I(h) \times y$, where h and y respectively denote the individual consumption of housing services and consumption good and where

$$I(h) = \begin{cases} 1, & h \geq 1 \\ 0, & h < 1. \end{cases} \tag{11}$$

Production of Y involves increasing returns at the city level, with $Y_j = A_j L_j^{1+\epsilon}$, where Y_j denotes output, A_j denotes (exogenous) total factor productivity (TFP), L_j denotes the size of city j's (endogenous) labor force and ϵ is a strictly positive parameter. Assume that the *individual* cost of living in city j (which includes expenditure in housing), denoted by c_j, is equal to θL_j^{γ}, where θ and are γ strictly positive parameters.

1. Based on the empirical work of Ciccone and Hall (American Economic Review, 1996) and others, what are the ranges for the values of ϵ and γ?

2. Why do large cities pay higher wages in equilibrium?

3. What <u>must</u> be true in a spatial equilibrium for any pair of cities with strictly positive populations? Specifically, write an equality involving the population sizes of these two cities. (Hint: you should come up with an expression for city-wide real wages first.)

4. What do you need to assume about ϵ and γ for this to be an actual spatial equilibrium?

5. Briefly list at least three possible economic mechanisms that generate city-wide increasing returns to scale.

6. Provide one microeconomic foundation for the relationship $Y_j = A_j L_j^{1+\epsilon}$ that is, briefly sketch an economic model that yields this relationship in equilibrium.

7. Provide one microeconomic foundation for the relationship $c_j = \theta L_j^{\gamma}$ that is, briefly sketch an economic model that yields this relationship in equilibrium.

8. Rank cities according to their TFP, i.e. $A_1 > A_2 > ... > A_j > ... > A_{J-1} > A_J$. What can you say about the ranking of the cities' populations $L_1, L_2, ...$ in equilibrium?

9. What is Zipf's law for cities?

10. Explain briefly how the model developed by Duranton (American Economic Review, 2007) can account for this statistical relationship.

11. To what extent does the model developed so far fail or succeeds in reproducing Zipf's law?

6. The economics of traffic and road congestion

Former exam question (Public Economics, January 2017).

In this problem we develop a simple framework to think about several issues related to traffic and how some familiar policies can address excessive congestion.

The set up is as follows:

- Consider a measure $N > 1$ of commuters who commute to the city center for work.
- Commuters may choose between two possible transportation modes: individual cars ('mode 1') or public transports ('mode 2').
- We denote the endogenous number of commuters who choose mode 1 in equilibrium by n (the number of commuters choosing mode 2 is thus $N - n$).
- The use of private cars generates congestion in the city center in the sense that commuting times of either mode is increasing in n:
  - The per-driver commuting time of mode 1 is equal to

    $$t_1(n) = n^\gamma / A,$$

    where $A > 0$ captures quality or capacity of roads and $\gamma > 0$ captures extend of congestion ('traffic jams');
  - The per-capita commuting time for bus users is

    $$t_2(n) = \tau + \alpha n^\beta,$$

    some α,$\tau > 0$, where $\beta > 0$ captures the effect of congestion on the speed of buses (for instance, some car drivers block crossroads when traffic is dense, slowing buses down).
- When choosing their commuting mode at the so-called 'decentralized equilibrium', commuters seek to minimize their commuting time treating t_1 and t_2 as parameters.

For simplicity, let $\alpha = 0$ so that $t_2(n) = \tau$ for sub-questions 1 to 9. We also impose $1 + \gamma < A\tau < N^\gamma$ (this is a technical assumption that makes the problem interesting and avoids a typology of cases).

1. Show that the number of commuters choosing mode 1 at the decentralized equilibrium is equal to

$$n_\star = (A\tau)^{1/\gamma}.$$

2. Is $n_\star$ increasing or decreasing in A, τ, and γ? Why (provide economic intuition)?

3. What is the aggregate commuting time in equilibrium (please denote it by $T_\star$)?

4. Consider next the optimal allocation. A planner would choose n in order to minimize the aggregate commuting time, defined as

$$T(n) \equiv nt_1(n) + (N-n)t_2(n).$$

   What is the optimal n (please denote it as n_0)?

5. Is n_0 increasing or decreasing in A, τ, and γ? Why (provide economic intuition)?

6. Is $n_\star$ larger or smaller than n_0? Why (provide economic intuition)?
   This finding is the first central result of this problem.

7. What is the optimal average commuting time (please denote it by T_0)?

8. Is $T_\star$ smaller or larger than T_0? Please provide the economic intuition for this result.

9. Consider the policy of building new lanes to extant roads or even adding new roads or building a new bridge to the center, i.e. consider an increase in A. What does this do to commuting times in equilibrium? Has congestion fallen?
   This finding, due to Downes and sometimes referred to as the *Fundamental Law of Congestion*, is the second central result of this problem.

10. **For the remainder of this problem and for simplicity, let** $\beta = \gamma = 1$ **and** $\alpha \in \left(0, \frac{\tau}{AN}\right)$ (the latter is a technical assumption made to avoid a typology of cases) **so that** $t_1(n) = n/A$ **and** $t_2(n) = \tau + \alpha n$**.**
    Show that the equilibrium fraction of commuters who chose the car is now equal to

$$n_\star = \frac{A}{1-\alpha A}\tau.$$

11. Show that the efficient allocation now requires the fraction of drivers to be equal to:

$$n_0 = \frac{A}{1-\alpha A}\frac{\tau - \alpha AN}{2}.$$

12. Is $n_\star$ larger or smaller than n_0? Why (provide economic intuition)?

13. According to your economic intuition, is $T_\star$ larger or smaller than T_0? Why?

14. **Bonus question.** Consider now increasing the number of lanes or building new roads, i.e. an increase in A. Our third central result, known as *Braess's Paradox*, says that

$$T_\star = \frac{n_\star^2}{A} + (N - n_\star)(\tau + \alpha n_\star)$$

may actually be *increasing* in A (it is indeed the case with our functional forms). Why can this result arise? What is the economic intuition?

7. Spatial disparities and EU structural funds

Former exam question (Public Economics 2018).

"Infrastructure investment has always been considered key for economic growth and has been one of the cornerstones of regional development strategies in the European Union (EU) and elsewhere. So intense has the focus on infrastructure been that formerly lagging regions have become leaders in transport infrastructure endowment. After 20 years of intensive European investment in transport infrastructure, Spain had the largest motorway network among the first 15 members of the EU, whereas Portugal leads in kms per GDP" (Crescenzi et al. 2016).[1]

Do such infrastructure investments reduce spatial disparities? Discuss critically in light of the models and empirical evidence.

[1] Crescenzi, Di Cataldo, and Rodríguez-Pose. 2016. Government infrastructure and the economic returns of transportation infrastructure investment in European Regions. Journal of Regional Science, 56(4), 555-82.