SOME INTERESTING RATIONAL CURVES ON K3 SURFACES

by Daniel CORAY and Several OTHERS

ABSTRACT. This is a mock paper to illustrate some features of the macros of *L'Enseignement Mathématique* [1]).

INTRODUCTION

The following assertion was made by B. Segre:

($\mathbb{S}$) *The general quartic surface F contains a finite number $c_h > 0$ of rational curves of degree $4h$ (for $h = 1, 2, 3, \ldots$).*

It is interesting to compare with a similar statement proved by Mumford and by Bogomolov (see [M-M]):

THEOREM (Bogomolov & Mumford). *Every algebraic K3 surface over $\mathbb{C}$ contains a singular rational curve and a pencil of singular elliptic curves.*

SCHOLIA. What happens in reality is somewhat surprising. We shall try to explain here why this is so.

SCHOLIUM 1. It is known that the Chow variety of space curves of degree m has an irreducible component $\mathcal{R}_m$, of dimension $4m$.

---

[1]) For more details on how to use this set of macros, the reader is referred to the specific instructions, which are at present only available in French.

1. About finiteness

Segre's heuristic starting point was that a complete linear system of arithmetic genus g on a smooth quartic is also of dimension g.

Lemma 1.1. *No K3 surface carries any one-dimensional algebraic system of irreducible rational curves.*

Proof. We denote by $p\colon \mathcal{J} \to X$ the first projection and observe that p is dominant, unless the family is constant. $\square$

Remark. Lemma 1.1 does not imply that a given K3 surface cannot contain infinitely many smooth rational curves.

2. About existence

Let $\mathfrak{Y} \subset V_\delta \times F$ be the universal Cartier divisor of the flat family.

Remarks. 1) The schemes parametrizing all curves of a given geometric genus have been well examined.

2) We can think of a smooth cubic surface as being $\mathbb{P}^2$ with six points blown up.

Definition 2.1. A *quartic* in $\mathbb{P}^3$ is a smooth surface of degree 4.

In the present paper we are especially interested in the case where $\lambda = 3$.

Proof of Theorem 4.1. As explained at the beginning of §3, we can work over an open set of reduced and irreducible curves. $\square$

Corollary 4.2′ *The smooth quartics in $\mathbb{P}^3$ carrying curves of degree 8 form a constructible set in $\mathcal{L}_K$.* $\square$

REFERENCES

[Ca] CAYLEY, A. On a new analytical representation of curves in space. *Quart. J. of Pure and App. Math. 3* (1860), 225–236 (*Collected Papers, vol. 4*, 446–455).

[De] DELIGNE, P. Le théorème de Noether. In : *SGA 7 II*, Lecture Notes in Mathematics no. 340, exposé 19, pp. 328–340. Springer (Berlin–Heidelberg–New York), 1973.

[Ha] HARTSHORNE, R. *Algebraic Geometry.* Springer (New York), 1977.

[M-M] MORI, S. and S. MUKAI. The uniruledness of the moduli space of curves of genus 11. In: *Lecture Notes in Maths 1016* (1983), 334–353.

Daniel Coray

Section de Mathématiques
2–4, rue du Lièvre
CH–1211 Genève 24
Switzerland
e-mail: coray@ibm.unige.ch

Several Others

Université de Genève
C.P. 240
CH–1211 Genève 24
Switzerland
e-mail: Several.Others@math.unige.ch