TRANSVERSE INTERSECTIONS
OF FOLIATIONS IN THREE-MANIFOLDS

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ABSTRACT. For two foliations of a three-manifold, we consider the question whether their transverse intersection is unique.

1. INTRODUCTION

In this paper we consider the transverse intersection of two foliations of a three-manifold ([3]).

THEOREM 1.1. *Let A be an element of* $\mathrm{SL}(2;\mathbf{Z})$ *such that* $|\mathrm{Tr}\,A| > 2$. *Then the intersection* $\mathcal{F}^u \cap h^*\mathcal{F}^s$ *is isotopic to the Anosov flow* $\mathcal{F}^u \cap \mathcal{F}^s$.

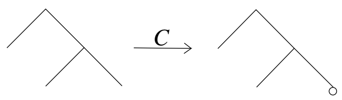

FIGURE 1

Our Theorem 1.1 is stronger than that in [1]. In §2, we construct several examples of multifoliations.

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†) We thank the second author for his help in providing this model file, whose mathematical content has been profoundly distorted *ad usum exempli*.

2. UNIT TANGENT BUNDLES OF THE HYPERBOLIC SURFACES

LEMMA 2.1. *If γ is a compact leaf of the intersection $\mathcal{F}^u \cap \mathcal{F}^s$, then the $\pi_1(M)$ orbit of the image $p(\widetilde{\gamma})$ of the lift $\widetilde{\gamma} \subset \widetilde{M}$ is discrete in $Q^u \times Q^s$.*

Proof. For any compact disk D of $Q^u \times Q^s$, one can take a lift $\widetilde{D}$ in $\widetilde{M}$. $\square$

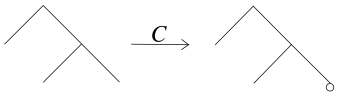

FIGURE 2

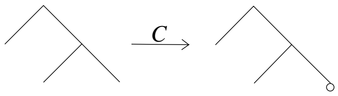

FIGURE 3

These foliations are transverse to each other and form a multifoliation. The intersections of several of their leaves are shown in Figure 5.

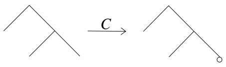

FIGURE 4

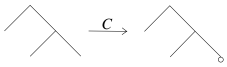

FIGURE 5

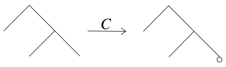

FIGURE 6

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