Talks

Workshop in Statistical Mechanics

February 2023

1 Xue-Mei Li

Title: What is fractional averaging?

Abstract : Multi-scale dynamics is prevalent and so is stochastic multi-scale systems. The aim is to the persistent effect of the fast motion on the slow motion and obtain the effective motion for the slow variable as the time scale separation parameter is taken to zero. This fundamental concept is at the heart of balancing the need of delicate more precise modelling and the need to extract information from the model. Yet studies until recently had been restricted on deterministic systems or systems with noise which are locally uncorrelated. Recently we embarked o nthe study of multi-scale stochastic differential equations driven by an autocorrelated stationary increment Gaussian process (fractional Brownian motions) with non-trivial multiplicative noise. Classical methods do not seem to apply, there are some very interesting and exciting new results.

2 Karol Kozlowski

Title: Universality in structures related to the XXZ spin-1/2 chain at low-temperature

Abstract : The quantum transfer matrix is an auxiliary tool allowing one to significantly simplify the problem of effectively calculating the the *per site* free energy as well as the correlation functions of a one dimensional quantum spin chain model at finite temperature. It is conjectured that certain universal features arising in the long-distance asymptotic behaviour of multi-point functions of critical one-dimensional quantum spin chains directly at zero temperature also manifest themselves on the level of the low-temperature behaviour of various quantities related with the associated quantum transfer matrix. In particular, if a given conformal field theory captures the long distance behaviour in the model et zero temperature, than the spectrum of this conformal field theory should arise in the low-temperature behaviour of the spectrum of the quantum transfer matrix.

In the case of the XXZ chain spin-1/2 chain, the quantum transfer matrix may be even chosen to be integrable, what allows one, in principle, to study the mentioned universality properties of its spectrum by means of the Bethe

Ansatz. In this talk, I will describe how the Bethe Ansatz approach can be put on rigorous grounds for the quantum transfer matrix subordinate to the XXZ chain. Further, I will explain how those results then allow one to access to the universal features of the spectrum of the quantum transfer matrix by showing that a subset thereof explicitly contains, in the low-temperature limit, the spectrum of the $c = 1$ free Boson conformal field theory.

3 Mikhail Basok

Title: Dimers on a Riemann surface and compactified free field

Abstract : In this talk I will speak about the dimer model considered on a general Riemann surface. The main goal is to show that height fluctuations associated with a sequence of dimer models on a given surface converge to the compactified free field in the scaling limit. I prove the convergence for a family of dimer graphs satisfying certain conditions which allow to use discrete complex analysis tools on them. To construct such a graph I first fix a locally flat metric with conical singularities on the Riemann surface. The graphs are then glued from planar templates satisfying certain geometric conditions (for example, being isoradial). Due to the recent result of Berestycki, Laslier and Ray, my result extends to a broad family of Temperley graphs satisfying some probabilistic conditions.

The paper is in preparation.

4 Martin Hairer

Title: Stochastic quantisation with fermions

5 Alice Contat and Nicolas Curien

Title: Critical core percolation on random graphs

Abstract : Motivated by the desire to construct large independent sets in random graphs, Karp and Sipser modified the usual greedy construction to yield an algorithm that outputs an independent set with a large cardinal called the Karp- Sipser core. When run on the Erdös-Rényi $G(n, c/n)$ random graph, this algorithm is optimal as long as $c <$ e. We will present the proof of a physics conjecture of Bauer and Golinelli (2002) stating that at criticality, the size of the Karp-Sipser core is of order $n^{3/5}$. Along the way we shall highlight the similarities and differences with the usual greedy algorithm and the k-core algorithm. Based on a joint work with Thomas Budzinski.

6 Dor Elboim

Title : Infinite cycles in the interchange process in five dimensions

Abstract : In the interchange process on a graph $G = (V, E)$, distinguished particles are placed on the vertices of G with independent Poisson clocks on the edges. When the clock of an edge rings, the two particles on the two sides of the edge interchange. In this way, a random permutation $\pi_\beta : V \to V$ is formed for any time $\beta > 0$. One of the main objects of study is the cycle structure of the random permutation and the emergence of long cycles. We prove the existence of infinite cycles in the interchange process on Z^d for all dimensions $d \geq 5$ and all large β, establishing a conjecture of Bálint Tóth from 1993 in these dimensions. In our proof, we study a self-interacting random walk called the cyclic time random walk. Using a multiscale induction we prove that it is diffusive and can be coupled with Brownian motion. One of the key ideas in the proof is establishing a local escape property which shows that the walk will quickly escape when it is entangled in its history in complicated ways.

This is a joint work with Allan Sly.

7 Colin Guillarmou

Title : Structure of conformal blocks of Liouville

Abstract : Abstract: I will explain how the conformal blocks can be constructed from the probabilistic methods of Liouville conformal field theory, and their global properties on Tecihmüller space. An important tool is a probabilistic représentation of the Virasoro algebra using Markov processes. Joint work with Baverez, Kupiainen, Rhodes, Vargas.

8 Eveliina Peltola

Title : On crossing probabilities in critical random-cluster models

Abtract: I will discuss exact solvability results (in a sense) for scaling limits of interface crossings in critical random-cluster models in the plane with various boundary conditions. The results are rigorous for the FK-Ising model, Bernoulli percolation, uniform spanning trees, and the spin-Ising model in appropriate setups. The scaling limit formulas describe structures in the corresponding boundary conformal field theory. (Based on joint works with Yu Feng, Mingchang Liu, and Hao Wu - all at Tsinghua University, China).

9 Avelio Sepulveda

Title: Exit sets of the Gaussian free field - an Overview

Abstract: In this talk, I will give an overview on the main results concerning the exit sets of the Gaussian free field (GFF). These random sets are, informally, the connected components of the level sets of the GFF that intersect the boundary. I will provide a formal definition of exit sets and discuss their uniqueness, as well as their relationship with real and imaginary chaos and scaling limit results. Additionally, I will discuss their main properties including monotonicity, multifractal spectrum, and phase transitions of their percolative properties This presentation is based on works with Juhan Aru, Titus Lupu, Christophe Garban, Lukas Schoug, Fredrik Viklund, and Wendelin Werner

10 Christophe Garban

Title: Surface law and charge rigidity for the Coulomb gas on Z^d

Abstract : I will start by introducing and motivating the (two-component) Coulomb gas on the d-dimensional lattice Z^d. I will then present some puzzling properties of the fluctuations of this Coulomb gas. The connection of this model with integer-valued fields and compact-valued spin systems will be emphasised through the talk. This is based on joint works with Avelio Sepúlveda and David Dereudre.

11 Mikhail Sodin

Title : Stationary planar point processes and analytic functions

Abstract: Here is a sample of questions I plan to discuss in my talk: 1. Why some stationary point processes generate stationary vector fields and some don't? How does a random counterpart of the Weierstrass zeta-function (from the theory of elliptic functions) help to understand this. 2. What is the size of fluctuations of line integrals of these random meromorphic functions? What is a signed-length of the curve and why it is relevant here? How many different limiting Gaussian fields a stationary process can have? 1, 2, maybe more? 3. Does there exist a random stationary entire function whose zero set is a Poisson point process?

Most of the talk will be based on joint work with Aron Wennman and Oren Yakir (https://arxiv.org/abs/2210.09882, https://arxiv.org/abs/2211.01312)

12 Piet Lammers

Title: The 2D XY model

Abstract : The 2D XY model has attracted attention of physicists and mathematicians for several decades. One way to understand this model is through its dual height function. Recent developments make it possible to show that the phase transitions of the two models coincide. The talk will highlight several connections between the two models and is based on arXiv:2301.06905 (Bijecting the BKT transition) and arXiv:2211.14365 (A dichotomy theory for height functions).

13 Romain Panis

Title: Translation invariant Gibbs measures and continuity for φ^4 via random tangled currents.

Abstract: In this talk I will present recent results obtained in joint work with Trishen Gunaratnam, Christoforos Panagiotis and Franco Severo concerning the study of Gibbs measures of the lattice φ_d^4 model on Z^d.

We prove that the set of translation invariant Gibbs measures for the φ_d^4 model on Z^d has at most two extremal measures at all temperature. We also give a sufficient condition to ensure that the set of all Gibbs measures is a singleton. As an application, we show that the spontaneous magnetisation of the nearest-neighbour φ_d^4 model on Z^d vanishes at criticality for $d \geq 3$. The analogous results were established for the Ising model in the seminal works of Aizenman, Duminil-Copin, and Sidoravicius (Comm. Math. Phys., 2015), and Raoufi (Ann. Prob., 2020) using the so-called random current representation introduced by Aizenman (Comm. Math. Phys., 1982). Our proof relies on a new corresponding stochastic geometric representation for the φ_d^4 model called the random tangled current representation.

14 Ellen Powell

Title: Brownian excursions, conformal loop ensembles and critical Liouville quantum gravity

Abstract: It was recently shown by Aïdékon and Da Silva how to construct a growth fragmentation process from a planar Brownian excursion. I will explain how this same growth fragmentation process arises in another setting: when one decorates a certain "critical Liouville quantum gravity random surface" with a conformal loop ensemble of parameter 4. This talk is based on joint work with Juhan Aru, Nina Holden and Xin Sun.

15 Antti Knowles

Title: Mobility edge for Erdös-Rényi graphs

Abstract: A disordered quantum Hamiltonian is mathematically modelled by a high-dimensional random operator. According to the general universality conjecture of disordered quantum Hamiltonians, the spectrum of such an operator can be in one of two phases: a localized phase where the eigenvectors are localized in physical space, and a delocalized phase where the eigenvectors are spread out throughout physical space. The interface of these phases is known as the mobility edge. A mobility edge is expected to occur in many models of quantum disorder, such as the Anderson model in three and higher dimensions. Nevertheless, its existence has proven remarkably impervious to rigorous analysis.

In this talk I consider a quantum Hamiltonian given by the adjacency matrix of the Erdös-Rényi graph, where the randomness arises from the random geometry of the graph. I show that, in a range of densities, the Erdös-Rényi graph exhibits a mobility edge. In each respective phase, the localization or delocalization estimate is quantitative with an optimal rate. Joint work with Johannes Alt and Raphael Ducatez.

16 Jose Padilla

Title: Analysis on the hypercube

Abstract: The set of vertices of a unit cube is called the hypercube. It consists of vectors with coordinates zeros and ones. In this talk we will discuss some series of analytic inequalities for subsets of the hypercube. We will focus on some recently obtained isoperimetric inequalities.

17 Alexander Glazman and Moritz Dober

Title: Phase diagram of Ashkin-Teller model

Abstract : The Ashkin-Teller model may be viewed as a pair of interacting Ising models. In the isotropic case, there are two coupling constants J,U where J describes the interaction in both Ising models and U the interaction between the two of them. We show that, depending on whether J¿U or J¡U, the model undergoes either a unique or two distinct phase transitions. This is joint work with Yacine Aoun and Alexander Glazman.

18 Béatrice de Tilière

Title: The dimer model on minimal graphs: the elliptic case and beyond.

Abstract : The dimer model represents the adsorption of diatomic molecules on the surface of a crystal. It is modeled through perfect matchings of a planar graph chosen with respect to the Boltzmann measure. When the graph is periodic, Kenyon, Okounkov and Sheffield show that the phase diagram is given by the spectral curve, which has the remarkable property of being Harnack. Another important result is the local expression obtained by Kenyon for one Gibbs measure when the underlying graph is isoradial and the model is critical. In a series of works with Cédric Boutillier (Sorbonne University) and David Cimasoni (University of Geneva), we extend these results in a unified framework. We consider the model on minimal graphs and prove an explicit correspondence with the set of Harnack curves; we also prove local formulas for the two parameter family of Gibbs measures.

19 Lucas D'Alimonte

Title: : Exact cube-root fluctuations in an area-constrained random walk model.

Abstract: This talk is devoted to the study of the behaviour of a (1+1)-dimensional model of random walk conditioned to enclose an area of order N^2. Such a conditioning forces the random walk to follow a globally concave trajectory; we study the local deviations of the walk from this concave shape. To this end, we study two quantities - the mean local roughness and the mean facet length - measuring the typical transversal and longitudinal fluctuations around the boundary of the convex hull of the random walk. Specifically, our main result is that the mean facet length (resp. the mean local roughness) is of order $\Theta(N^{2/3})$ (resp. $\Theta(N^{1/3})$). This model is intended to be a simple model for the interface of a two-dimensional statistical mechanics model (such as the Ising model) in the phase separation setting. Joint work with Romain Panis.