Multiscale analysis of a transmission problem

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LAGA - Université Paris 13

Workshop on multiscale methods for deterministic and stochastic dynamics

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Collaboration with Martin Gander, Véronique Martin
Philippe d'Anfray, Juliette Ryan et Michel Borrel, Oana Ciobanu, ONERA

My modest multiscale promenade.

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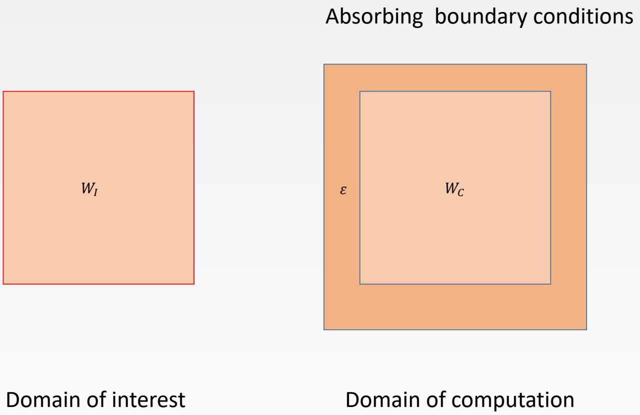
Summary

Absorbing boundary conditions for parabolic equations

LH +M. Schatzmann +J. Rauch 86-95



The convection-diffusion equation

$$\mathcal{L}(u) := u_t - \nu\Delta u + a\partial_1 u + \boldsymbol{b}\cdot\boldsymbol{\nabla} u + cu = f$$
$$u(\cdot,0) = u_0$$

$$\nu > 0,\; a > 0, b \in \mathbb{R}^n, c > 0.$$

$$\mathcal{B}_n = \left(-\frac{\nu}{a}\right)^{n-1}\mathcal{B}_1^n, \quad \mathcal{B}_1 = \partial_t + a\partial_1 + cI.$$

v_n solution with $\mathcal{B}_n v_n = 0$ on the exterior boundary of the layer,

$$\|u - v_n\|_{L^2(\Omega_l)} \leq \frac{1}{\sqrt{2a}}\nu^{2n+1/2}e^{-\frac{a}{\nu}\varepsilon}\|\mathcal{L}^{2n}u\|_{L^2(\Omega_l)}$$

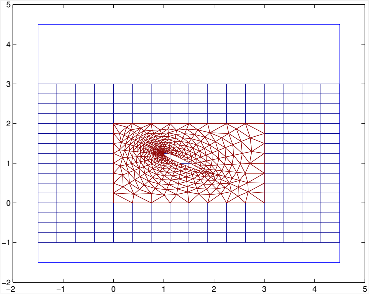
If $\nu \ll 1$, we choose $\varepsilon \approx Ch$,and get

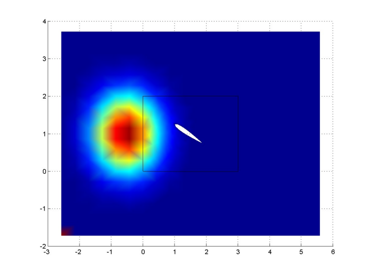
$$\|u - v_n\|_{L^2(\Omega_l)} \leq \frac{1}{\sqrt{2a}}\nu^{2n+1/2}e^{-2CPe}\|\mathcal{L}^{2n}u\|_{L^2(\Omega_l)}$$

$$Pe = \frac{ah}{2\nu}$$

Towards coupling

Encapsulated codes. P. d'Anfray and J. Ryan. 2002.





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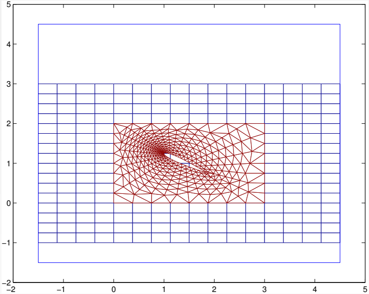

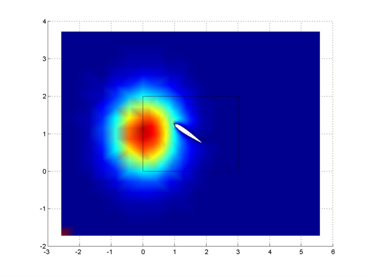

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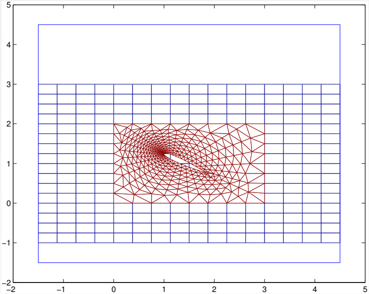

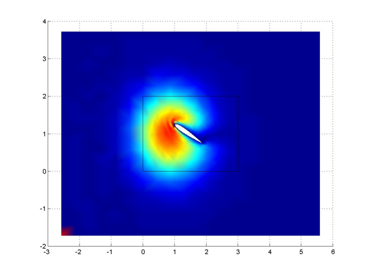

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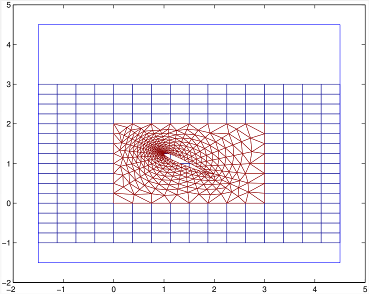

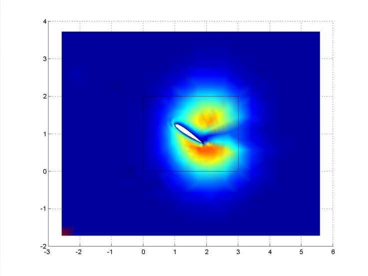

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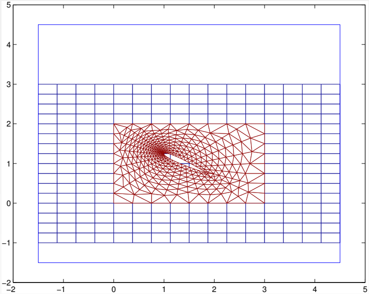

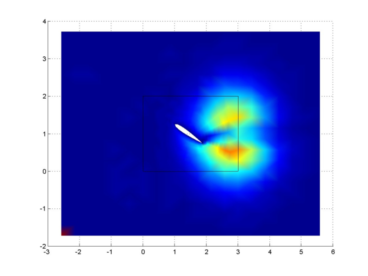

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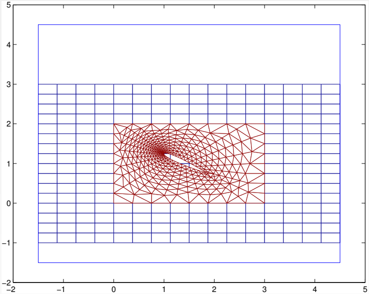

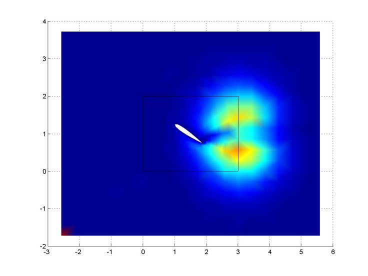

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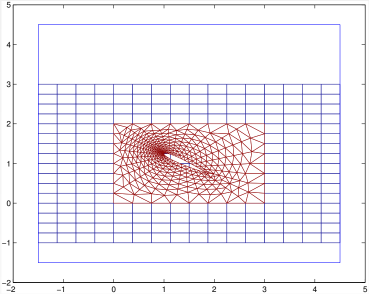

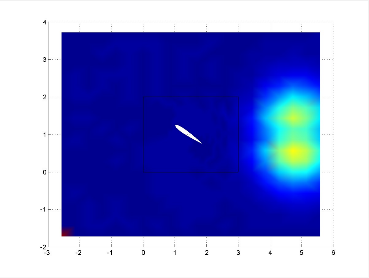

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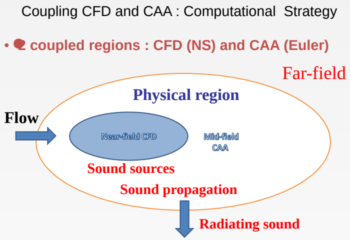
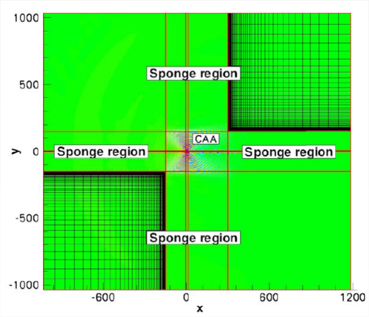
Couple "for good" Euler and Navier-Stokes. P. Borrel and J. Ryan. 2006-11. Thèse Oana CIOBANU.





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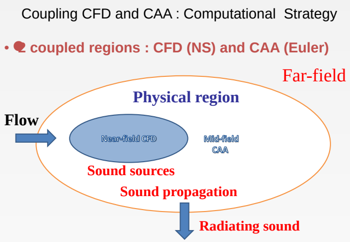

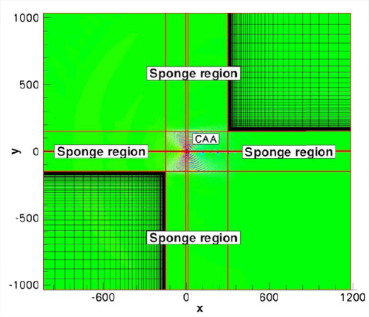

Towards coupling

- Instead of having $\mathcal{B}_n v$ on the boundary, use it in a layer. Eric Dubach's thesis, stationary AD equation. 1993.
- Beyond Dubach's work, write an algorithm which improves the error at each iterate. M. Gander, C. Japhet, V. Martin, 2002-2016.
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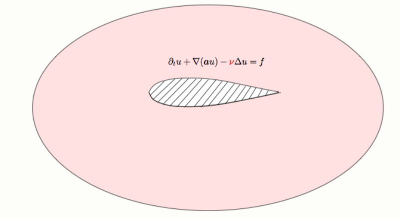
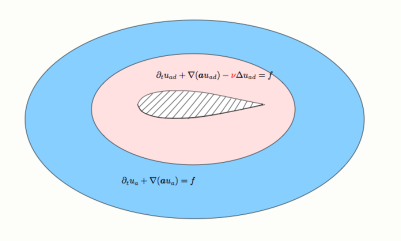
Summary

What is the question ?





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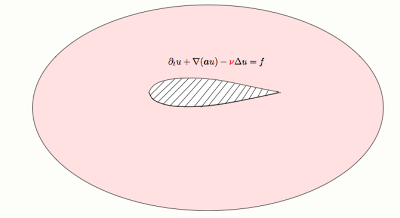

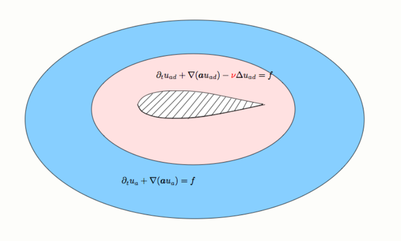

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Some previous answers for the steady-state : transmission conditions

- ☑ **Fabio Gastaldi and Alfio Quarteroni.** (1989). *On the Coupling of Hyperbolic and Parabolic Systems : Analytical and Numerical Approach*, Applied Numerical Mathematics,
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Summary

Problem $a > 0$

$$\mathcal{L}_{ad} u := -\nu \partial_{xx} u + \underbrace{\partial_t u + a\partial_x u + cu}_{\mathcal{L}_a u} = f \text{ in } \Omega,$$

$$u(-L_1, \cdot) = g_1 \text{ L.B.C.} \quad \mathcal{L}_a u(L_2, \cdot) = 0 \text{ R.B.C.}$$

$$u(x, 0) = u_0 \text{ I.C.}$$

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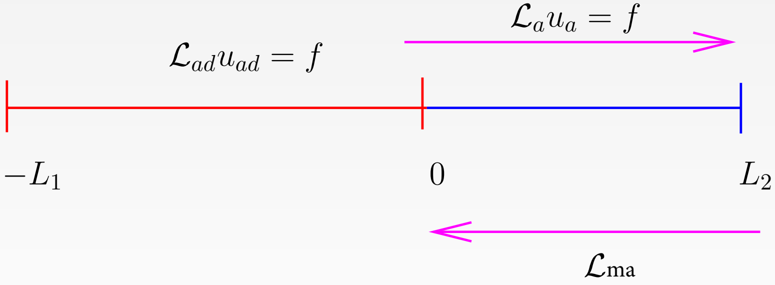
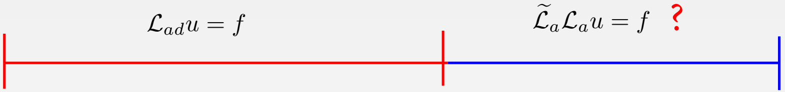


Factorisation $a > 0$

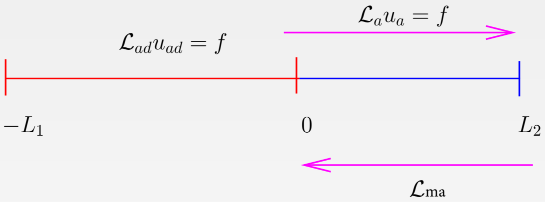


Factorisation $a > 0$

$$\mathcal{L}_a := \partial_t + a\partial_x + c, \quad \mathcal{L}_{ad} = \mathcal{L}_{ma}\mathcal{L}_a + \frac{\nu}{a^2}\mathcal{R}$$

$$\mathcal{L}_{ma} = -\frac{\nu}{a^2}(a\partial_x - \partial_t - c - \frac{a^2}{\nu}), \quad \mathcal{R} = (\partial_t + c)^2.$$

J.P. Lohéac, F. Nataf and M. Schatzman , Parabolic Approximations of the Convection-Diffusion Equation, Math. of Comp., 60 (2002), p.515-530, 1993.



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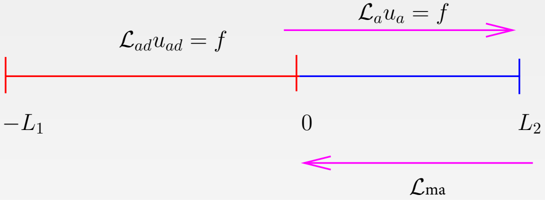

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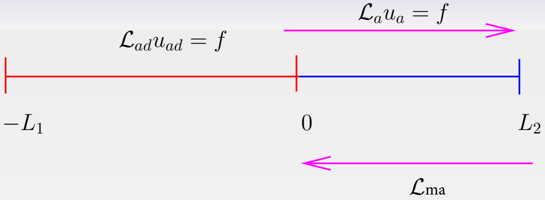

$$\mathcal{L}_{ad} = \mathcal{L}_{ma}\mathcal{L}_a + \frac{\nu}{a^2}\mathcal{R}$$

$$\begin{aligned} &\mathcal{L}_{ad} u_{ad} = f \text{ in } \Omega_1 \\ &u_{ad}(\cdot, 0) = u_0 \\ &u_{ad}(-L_1, \cdot) = g, \\ &\mathcal{L}_a u_{ad}(0, \cdot) = u_{ma}(0, \cdot), \end{aligned}$$

$$\begin{aligned} &\mathcal{L}_{ma} u_{ma} = f - \tfrac{\nu}{a^2}\mathcal{R} u_a \text{ in } \Omega_2 \\ &u_{ma}(\cdot, 0) = w_0 := f(\cdot, 0) + \nu u_0''. \\ &u_{ma}(L_2, \cdot) = 0. \end{aligned}$$

$$\begin{aligned} &\mathcal{L}_a u_a = f \text{ in } \Omega_2 \\ &u_a(\cdot, 0) = u_0 \\ &u_a(0, \cdot) = u(0, \cdot), \end{aligned}$$

Iterative coupling $a > 0$

$$\mathcal{L}_{ad} u_{ad} = f \text{ in } \Omega_1$$
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$$\mathcal{L}_a u_a = f \text{ in } \Omega_2$$
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$$u^k_{ad}(-L_1, \cdot) = g,$$
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$$\mathcal{L}_a u^k_a = f \text{ in } \Omega_2$$
$$u^k_a(\cdot, 0) = u_0$$
$$u^k_a(0, \cdot) = u^{k-1}_{ad}(0, \cdot).$$

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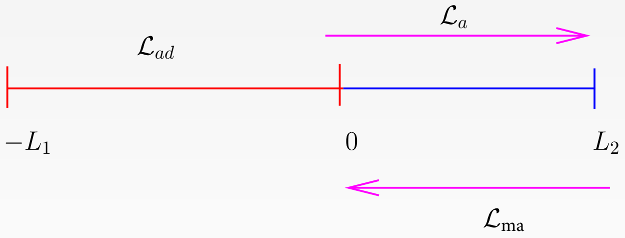
$$\begin{aligned} \mathcal{L}_{ad} u_{ad}^k &= f \text{ in } \Omega_1 \\ u_{ad}^k(\cdot, 0) &= u_0 \\ u_{ad}^k(-L_1, \cdot) &= g, \\ \mathcal{L}_a u_{ad}^k(0, \cdot) &= u_{ma}^k(0, \cdot), \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{ma} u_{ma}^k &= f - \frac{\nu}{a^2} \mathcal{R} u_a^k \text{ in } \Omega_2 \\ u_{ma}^k(\cdot, 0) &= f(\cdot, 0) + \nu u_0'' \\ u_{ma}(L_2, \cdot) &= 0. \end{aligned}$$

$$\begin{aligned} \mathcal{L}_a u_a^k &= f \text{ in } \Omega_2 \\ u_a^k(\cdot, 0) &= u_0 \\ u_a^k(0, \cdot) &= u_{ad}^{k-1}(0, \cdot). \end{aligned}$$

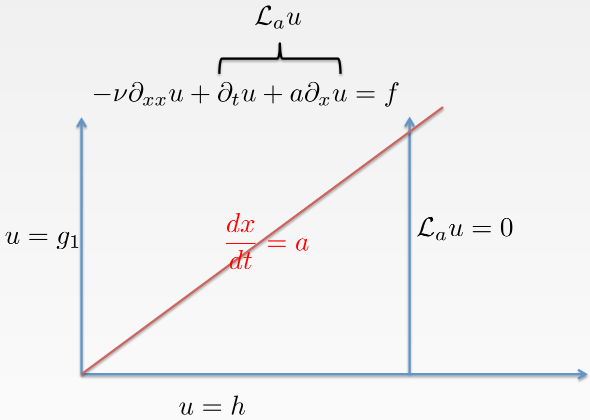


Summary

References

Métivier, G. (2012). Small Viscosity and Boundary Layer Methods : Theory, Stability Analysis, and Applications. Springer Science and Business Media.
Shih, S. D. (2007). Internal layers of parabolic singularly perturbed problems. ZAMM Journal of Applied Mathematics and Mechanics, 87(11-12), 831-844.

The advection-diffusion equation



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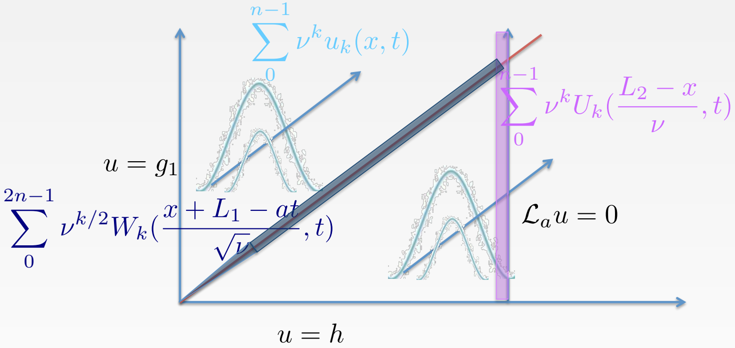
$$-\nu\partial_{xx}u + \partial_t u + a\partial_x u = f$$



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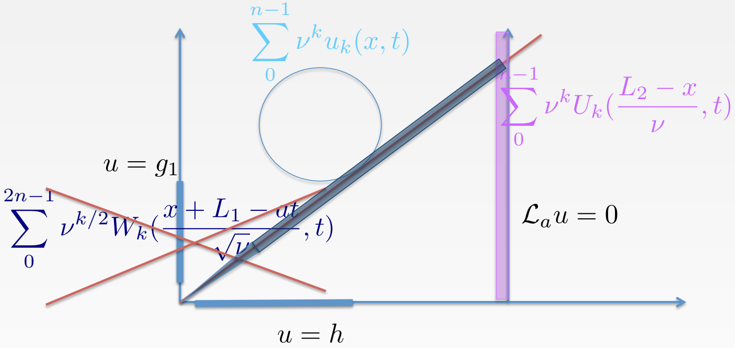

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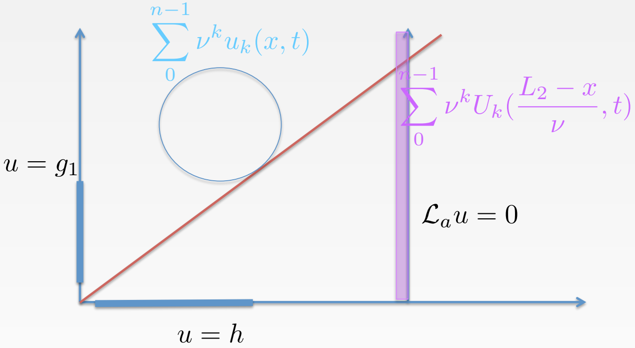
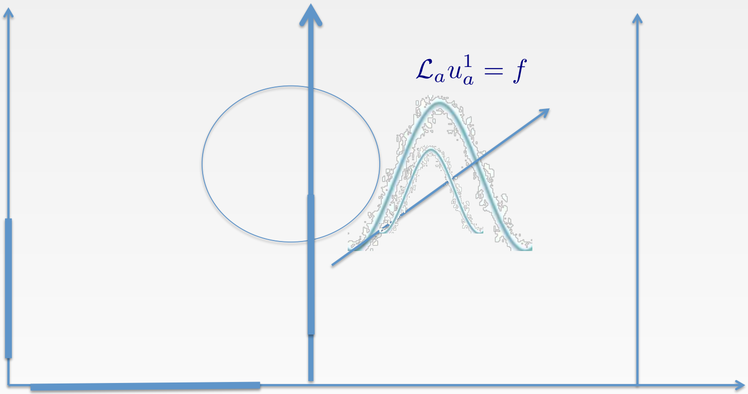


The factorization algorithm



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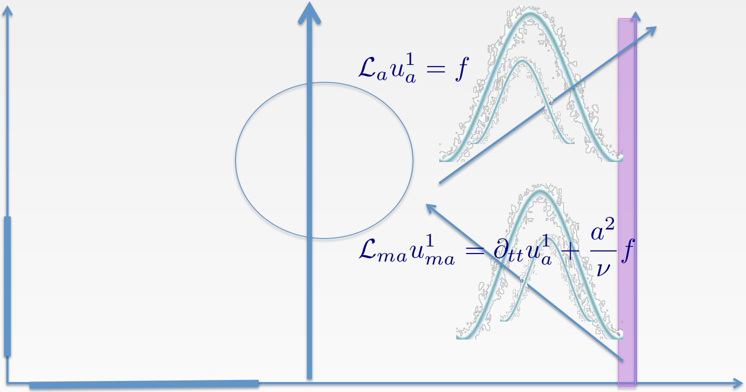

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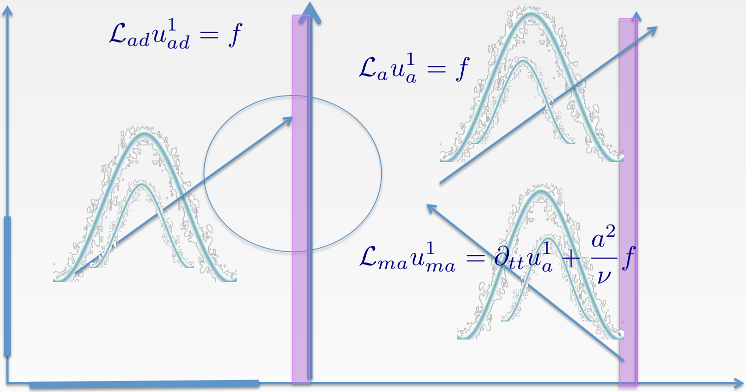

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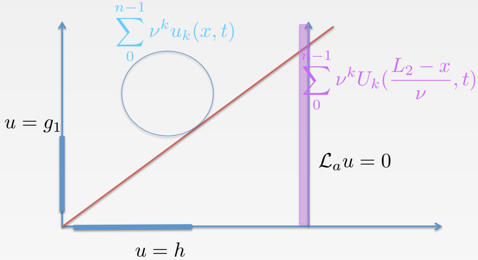
$-\nu\partial_{xx}u + \partial_t u + a\partial_x u = f$



$$u(x,t) = \underbrace{\sum_{j\geq 0} \nu^j u_j(x,t)}_{u^{out}} + \underbrace{\sum_{j\geq 2} \nu^j U_j^*(\frac{L_2 - x}{\nu}, t)}_{u^{in}}.$$

Each term in the *outer expansion* u^{out} is solution of a transport equation,

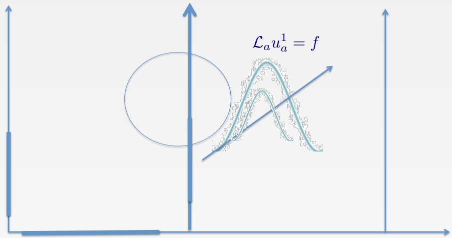
$$\mathcal{L}_a u_0 = f, \qquad u_0(x,0) = h(x), \qquad u_0(-L_1,t) = g_1(t), \qquad (1)$$

$$\mathcal{L}_a u_j = \partial_x^2 u_{j-1}, \qquad u_j(x,0) = 0, \qquad u_j(-L_1,t) = 0, \quad j \geq 1. \qquad (2)$$

The first non vanishing term in the *inner expansion* u^{in} is

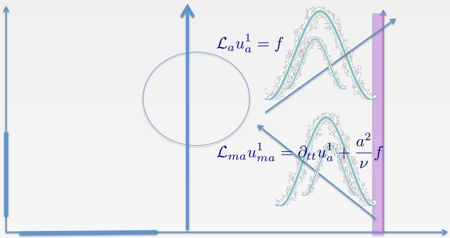
$$U_2^*(y,t) = -\frac{1}{a^2}\partial_x^2 u_0(L_2,t)e^{-ay}.$$

The algorithm, first advection steps



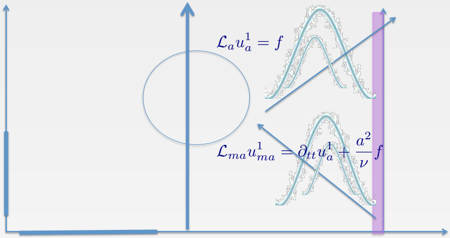
$$\mathcal{L}_a u_a^1 := \partial_t u_a^1 + a\partial_x u_a^1 = f, \;\; u_a^1(0,\cdot) = g^0.$$

The algorithm, first advection steps



$$\mathcal{L}_{ma} u_{ma}^1 := \partial_t u_{ma}^1 - a\partial_x u_{ma}^1 + \frac{a^2}{\nu} u_{ma}^1 = \mathcal{R} u_a^1 + \frac{a^2}{\nu} f, \;\; u_{ma}^1(0,t) = 0.$$

The algorithm, first advection steps



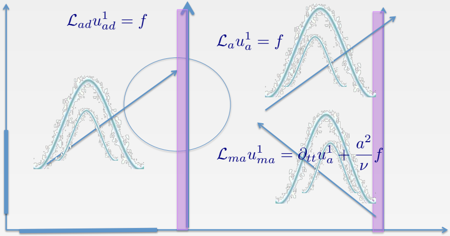
$$\mathcal{L}_{ma} u_{ma}^1 := \partial_t u_{ma}^1 - a\partial_x u_{ma}^1 + \frac{a^2}{\nu} u_{ma}^1 = \mathcal{R} u_a^1 + \frac{a^2}{\nu} f, \;\; u_{ma}^1(0,t) = 0.$$

$$u_{ma}^{1,out}(x,t) + u_{ma}^{1,in}(x,t) = \sum_{j\geq 0} \nu^j u_{ma,j}^1(x,t) + \sum_{j\geq 1} \nu^j U_{ma,j}^{1,*}\left(\frac{L_2 - x}{\nu}, t\right). \quad (3)$$

$$u_{ma,0}^1 = f, \quad u_{ma,j}^1 = \left(-\frac{\mathcal{L}_{ma}^0}{a^2}\right)^{j-1} \partial_x^2 u_a^1 \quad \text{for } j \geq 1, \quad (4)$$

with $\mathcal{L}_{ma}^0 = \partial_t - a\partial_x$.

The algorithm, advection diffusion step 1



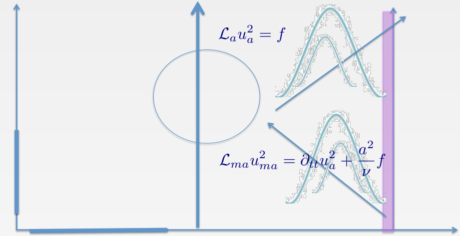
$$\mathcal{L}_{ad} u^1_{ad} = f, \ \mathcal{L}_a u^1_{ad}(0,\cdot) = u^1_{ma}(0,\cdot).$$

$$u^{out}(x,t) + u^{1,in}_{ad}(x,t) = \sum_{j\geq 0} \nu^j u_j(x,t) + \sum_{j\geq 2} \nu^j U^{1,*}_{ad,j}(\frac{-x}{\nu}, t). \qquad (5)$$

The first non vanishing term in the inner expansion $u^{1,in}_{ad}$ is

$$U^{1,*}_{ad,2}(y,t) = -\frac{1}{a^4} \partial_{tt} (u_0 - u^1_a)(0,t) e^{-ay}. \qquad (6)$$

The algorithm, second advection steps

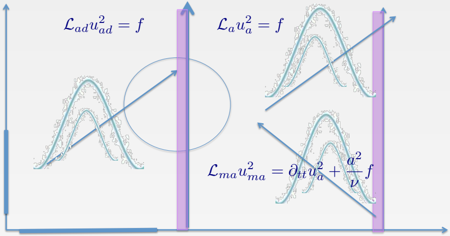


$$u_{ma}^{2,out}(x,t) + u_{ma}^{2,in}(x,t) = \sum_{j\geq 0} \nu^j u_{ma,j}^2(x,t) + \sum_{j\geq 1} \nu^j U_{ma,j}^{2,*}\left(\frac{L_2 - x}{\nu}, t\right). \quad (7)$$

The first terms in the *outer expansion* $u_{ma}^{2,out}$ are given at $x = 0$ by

$$u_{ma,0}^2 = f, \quad u_{ma,1}^2 = \mathcal{L}_a u_1, \quad u_{ma,2}^2 = \mathcal{L}_a u_2,$$
$$u_{ma,3}^2 = \mathcal{L}_a u_3 - \frac{1}{a^6}\partial_t^4\left(u_0 - g_{ad}^0\right) - \frac{1}{a^4}\partial_t^2\partial_x^2 u_0. \quad (8)$$

The algorithm, advection diffusion step 2



$$u^{out}(x,t) + u^{2,in}_{ad}(x,t) = \sum_{j\geq 0} \nu^j u_j(x,t) + \sum_{j\geq 4} \nu^j U^{2,*}_{ad,j}(\frac{-x}{\nu}, t). \qquad (9)$$

The first non vanishing term in the inner expansion $u^{2,in}_{ad}$ is

$$U^{2,*}_{ad,4}(y,\cdot) = -\frac{1}{a^8}\partial_{tt}(\partial_{tt}(u_0(0,\cdot) - g^0_{ad}) + a^2\partial^2_x u_0(0,\cdot))e^{-ay}. \qquad (10)$$

Summary

$$u(x,t) \approx u^{out}(x,t) + u^{in}(x,t) = \sum_{j\geq 0} \nu^j u_j(x,t) + \sum_{j\geq 2} \nu^j U_j^*(\frac{L_2 - x}{\nu}, t), \;\; x \in \Omega,$$

$$u_a^1(x,t) = u_0(x,t), \; x \in \Omega_2,$$

$$u_a^2(x,t) \approx u_a^{out}(x,t) = u_0(x,t) + \sum_{j\geq 1} \nu^j u_{a,j}^2(x,t), \;\; x \in \Omega_2,$$

$$u_{ad}^1(x,t) \approx u^{out}(x,t) + u_{ad}^{1,in}(x,t) = \sum_{j\geq 0} \nu^j u_j(x,t) + \sum_{j\geq 2} \nu^j U_{ad,j}^{1,*}(\frac{-x}{\nu}, t), \;\; x \in \Omega_1,$$

$$u_{ad}^2(x,t) \approx u^{out}(x,t) + u_{ad}^{2,in}(x,t) = \sum_{j\geq 0} \nu^j u_j(x,t) + \sum_{j\geq 4} \nu^j U_{ad,j}^{2,*}(\frac{-x}{\nu}, t), \;\; x \in \Omega_1,$$

with

$$\begin{aligned} U_2^*(y,\cdot) &= -\frac{1}{a^2}\partial_x^2 u_0(L_2,t)e^{-ay}, \\ U_{ad,2}^{1,*}(y,\cdot) &= -\frac{1}{a^4}\partial_{tt}\,(u_0 - u_a^1)(0,t)e^{-ay}, \qquad (11) \\ U_{ad,4}^{2,*}(y,\cdot) &= -\frac{1}{a^8}\partial_{tt}(\partial_{tt}\,(u_0(0,\cdot) - g_{ad}^0) + a^2\partial_x^2 u_0(0,\cdot))e^{-ay}. \end{aligned}$$

Error estimates

$$\|u - u_a^2\|_{L^2(\Omega_2\times(0,T))} \sim \nu\|e_{a,1}^2\|_{L^2(\Omega_2\times(0,T))}, \tag{12}$$

$$\|u - u_{ad}^1\|_{L^2(\Omega_1\times(0,T))} \sim \frac{\nu^{\frac{5}{2}}}{\sqrt{2a^9}}\|\partial_{tt}\left(u_0(0,\cdot) - g_{ad}^0\right)\|_{L^2(0,T)}, \tag{13}$$

$$\|u - u_{ad}^2\|_{L^2(\Omega_1\times(0,T))} \sim \frac{\nu^{\frac{9}{2}}}{\sqrt{2a^{17}}}\|\partial_{tt}(\partial_{tt}\left(u_0(0,\cdot) - g_{ad}^0\right) + a^2\partial_x^2 u_0(0,\cdot))\|_{L^2(0,T)} \tag{14}$$

with $e_{a,1}^2$ defined by

$$\mathcal{L}_a e_{a,1}^2 = \partial_x^2 u_0, \quad e_{a,1}^2(\cdot,0) = 0, \quad e_{a,1}^2(0,\cdot) = 0. \tag{15}$$

Negative advection

$$\|u - u_a^1\|_{L^2_{x,t}} \sim \nu \|u_1\|_{L^2_{x,t}}, \quad \|u - u_{ad}\|_{L^2_{x,t}} \sim \nu^2 \|u_2 - u_{ad,2}\|_{L^2_{x,t}},$$

Comparison

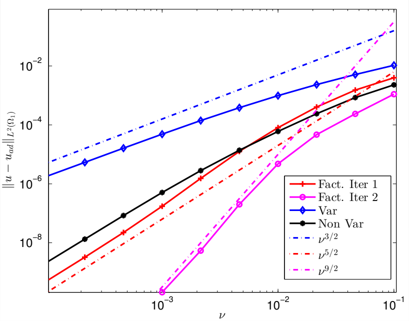
$\Omega := (-1,1)$, $\Omega_1 = (-1,0)$, $\Omega_2 = (0,1)$.

$$u_t + au_x - \nu u_{xx} + cu = f$$

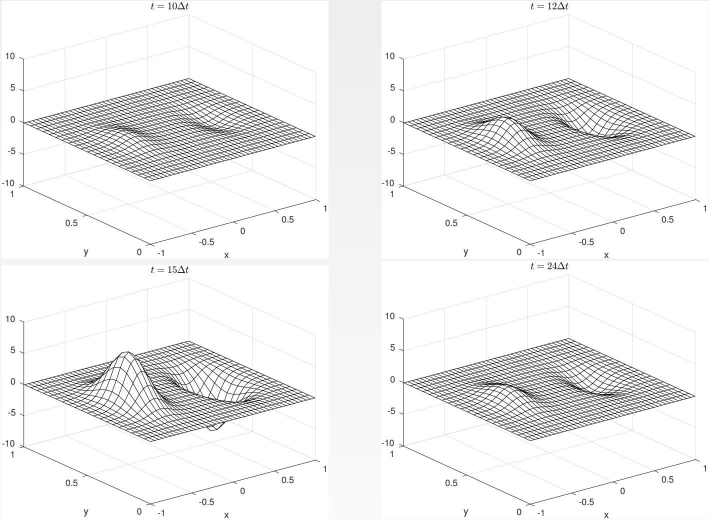
$a = 1,\ c = 1,\ T = 0.5$ and varying ν. I.C : $h(x) = e^{-100(x+0.5)^2}$,

$f(x,t) = f_1(t)f_2(x)$, $f_1(t) = 10\sin^4(4\pi(t-0.05))\chi_{t>0.05}$ $f_2(x) = -e^{-30(x-0.5)^2} +$

AD : Crank-Nicolson scheme, A : implicit upwind,
$\Delta t = \Delta x = 1.5625\,10^{-5}$.



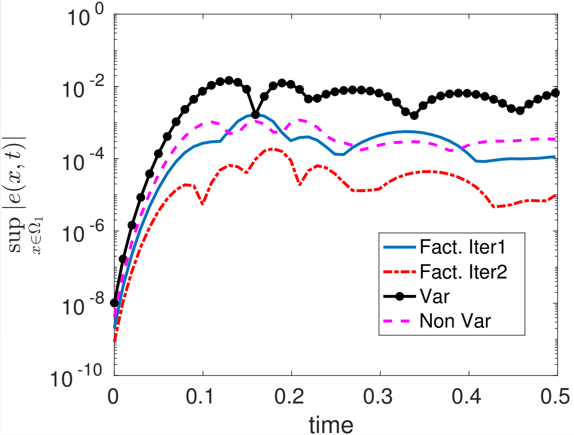
Extension to 2-D



Snapshots of the right hand side function at times $t = 10\Delta t$, $12\Delta t$, $15\Delta t$ and $24\Delta t$.

Extension to 2-D

Comparison

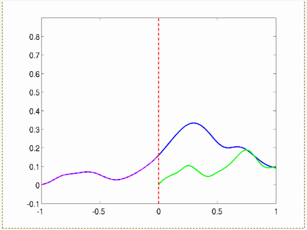


Error

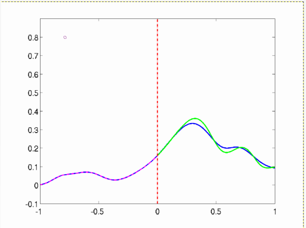
$\sup_{x\in\Omega_1} |e(x,t)|$ as a function of time t, where e stands for the error between the viscous solution and the coupled one in the viscous domain

Summary

- Eric Dubach *Contribution à la Résolution des Équations fluides en domaine non borné*. Thèse Université Paris 13,1993. **Coupling 1 and 2D steady**
- M. Gander, LH, C. Japhet and V. Martin. *Viscous Problems with Inviscid Approximations in Subregions : a New Approach Based on Operator Factorization*. ESAIM Proc., 27, 2009, pp. 272–288. **Steady 1D**
- M. Gander, LH, V. Martin, *A new Algorithm Based on Factorization for Heterogeneous Domain Decomposition*. Numer. Algorithms, 73(1), pp 167-195, 2016. **Evolution 1D**.
- M. Gander, LH, V. Martin, *Multiscale analysis of heterogeneous domain decomposition methods for time-dependent advection reaction diffusion problems*. JCAM 344, pp 904-924, 2018. **Multiscale expansion**.

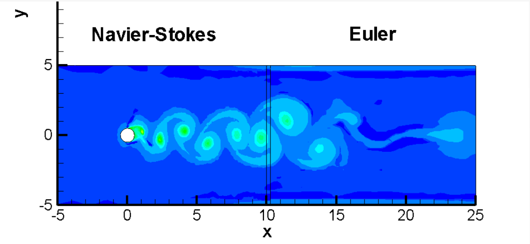


First iterate



Second iterate

• P. D'Anfray , L. Halpern and J. Ryan. *New trends in coupled simulations featuring domain decomposition and metacomputing*. M2AN Vol. 36, # 5, pp 953-970, September/October 2002
• M. Borrel, LH , J. Ryan, *Euler - Navier-Stokes coupling for aeroacoustics problems*. Computational Fluid Dynamics 2010 : Proceedings of the Sixth International Conference on Computational Fluid Dynamics, ICCFD6, St Petersburg, Russia.**Schwarz waveform relaxation**.
• M. Borrel, LH, J. Ryan, *Euler/Navier-Stokes coupling for multiscale aeroacoustics problems*. 20th AIAA computational fluid dynamics conference, june 2011, Hawai. AIAA 2011-3047. **"Full overlap" Schwarz waveform relaxation or chimera method**
• J. Ryan, LH, M. Borrel, *Domain decomposition vs. overset Chimera grid approaches for coupling CFD and CAA*. ICCFD7, Hawai, 2012. **Mixing layer**



Summary

Conclusions

- ☑ New coupling algorithm viscous/inviscid in 1D, taking the boundary conditions into account.
- ☑ Multiscale analysis gives the exact error.
- ☑ Industrial coupling code Euler/Navier-Stokes with Schwarz Waveform relaxation, discontinuous Galerkin and RK4 and nonconformal coupling.

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