



## Graphene nanocones and Pascal matrices

A surprising connection with famous models in combinatorics (plane partitions, lozenge tilings, dense loops on cylinder)

$$H_2 = \begin{array}{c|cccc} x+y & 0 & 1 & & \\ \hline 0 & 0 & 1 & y & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ & x & 1 & 0 & 0 & 1 \\ \hline & 0 & 0 & 0 & 0 & 1 & y \\ & 1 & & 1 & 0 & 1 & \\ & & 0 & & 1 & 0 & 1 \\ & & & 1 & & 1 & 0 & 1 \\ & & & & x & & 1 & 0 \end{array}$$

Adjacency matrix of a nanocone (size  $n^2$ )

### DETERMINANT?

$$Q_5 = \begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 6 & 10 & 15 \\ \hline 1 & 4 & 10 & 20 & 35 \\ 1 & 5 & 15 & 35 & 70 \end{array}$$

Pascal Matrix (size  $n$ )

$$\det H_n(e^{-i\theta}, e^{i\theta}) = e^{-i(n+1)\theta} \det(Q_n + e^{2i\theta})$$

$n$	$\theta = 0$	$\pi/6$	$\pi/3$	$\pi/2$	$\pi/4$
2	20	$3^2\sqrt{3}$	7	0	$8\sqrt{2}$
3	132	$10^2$	42	$2^4$	70
4	1452	$25^2\sqrt{3}$	429	0	$526\sqrt{2}$
5	26741	$140^2$	7436	$7^4$	13167
6	826540	$588^2\sqrt{3}$	218348	0	$280772\sqrt{2}$