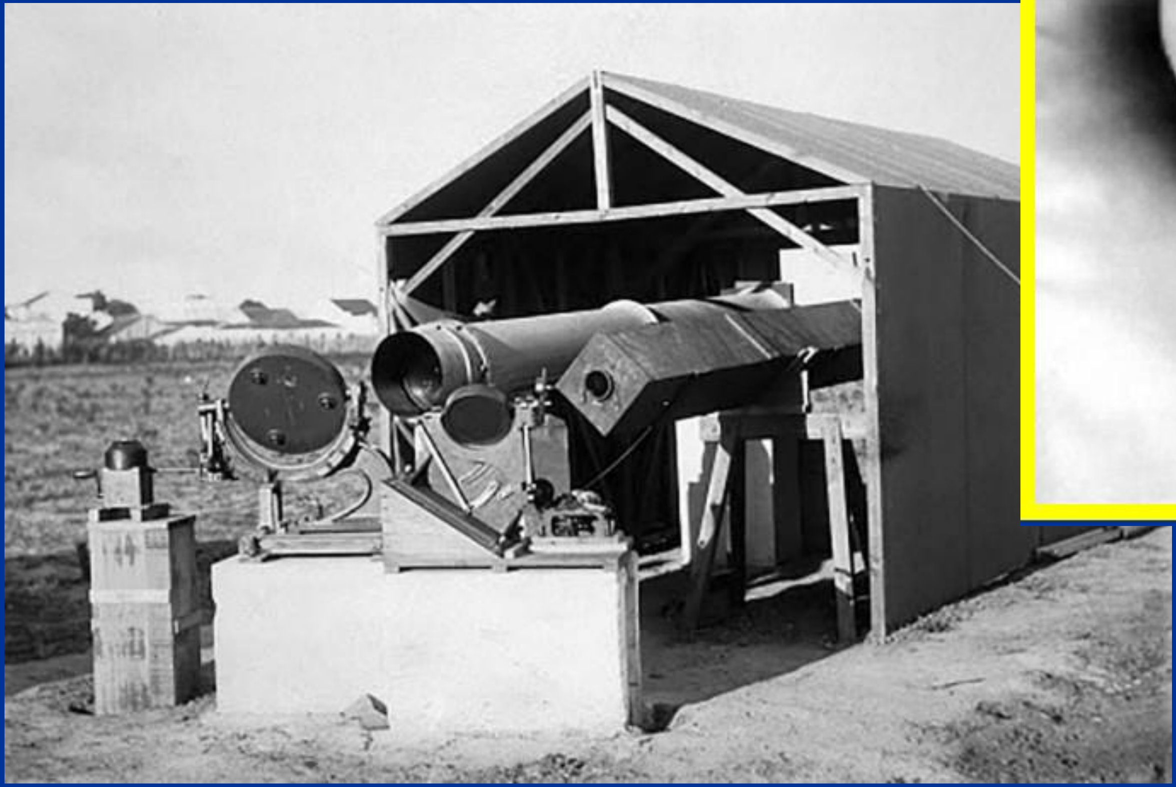
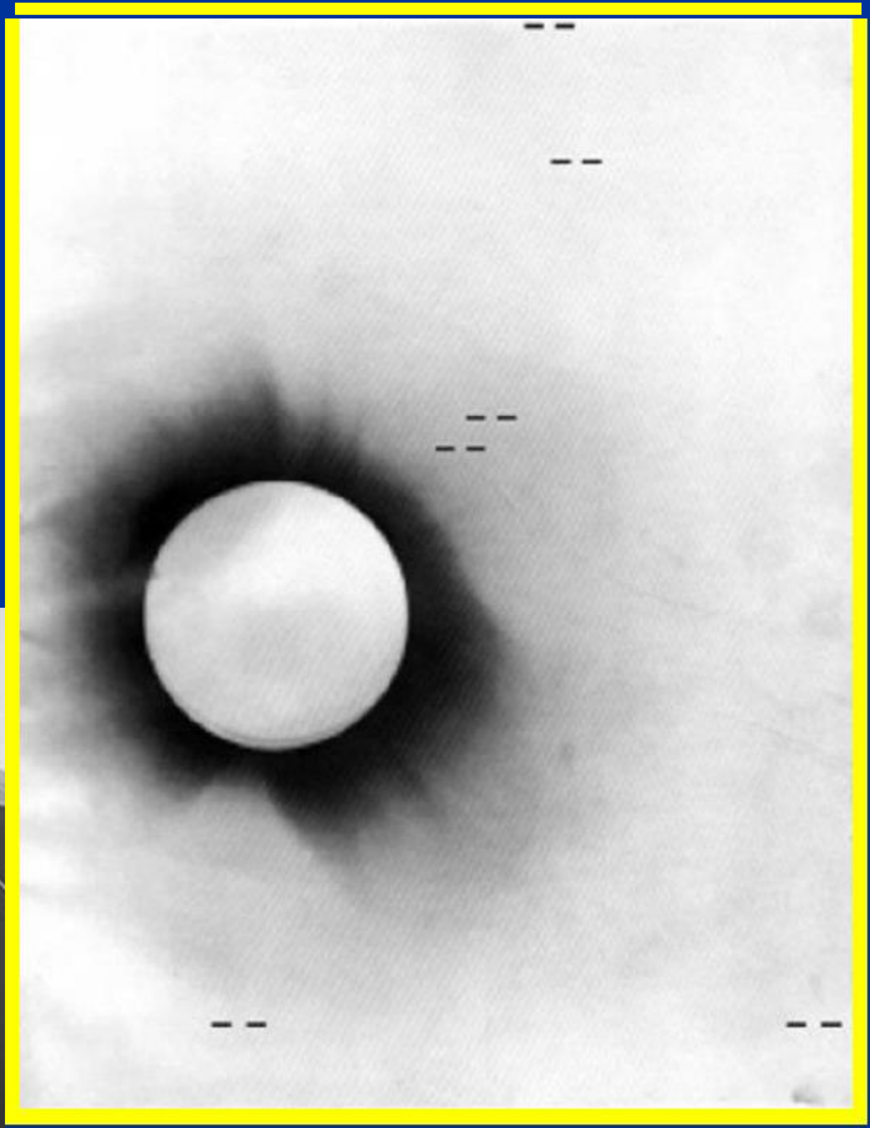
Solar eclipse of May 29 1919 in the island of Principe





EDDINGTON:
investment in Physics
and burst of results.

1923

THE MATHEMATICAL THEORY
OF
RELATIVITY

BY

A. S. EDDINGTON, M.A., M.Sc., F.R.S.

PLUMIAN PROFESSOR OF ASTRONOMY AND EXPERIMENTAL PHILOSOPHY IN THE UNIVERSITY OF CAMBRIDGE

THE
INTERNAL CONSTITUTION
OF THE STARS

BY

A. S. EDDINGTON

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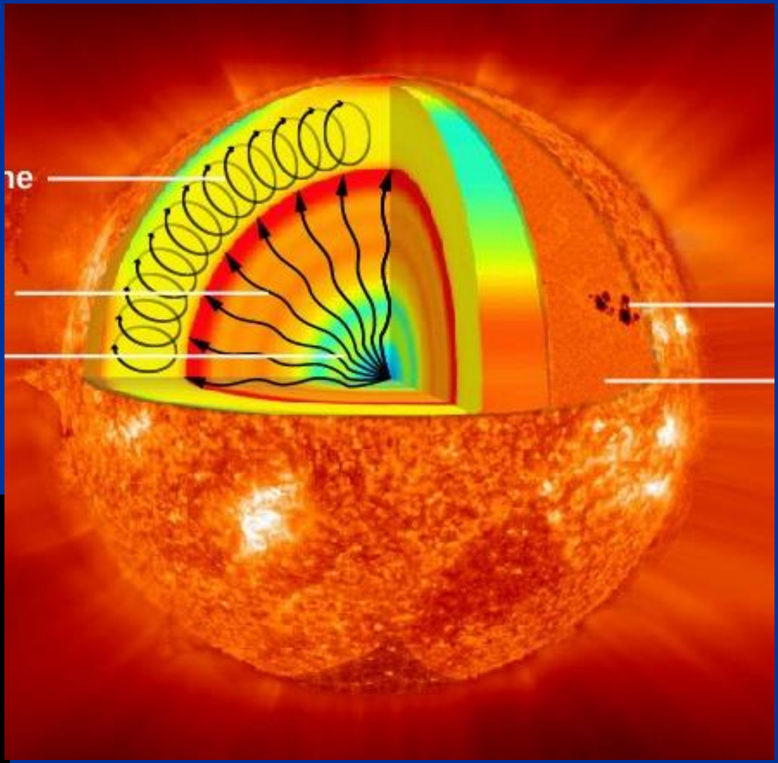
Plumian Professor of Astronomy in the University of Cambridge

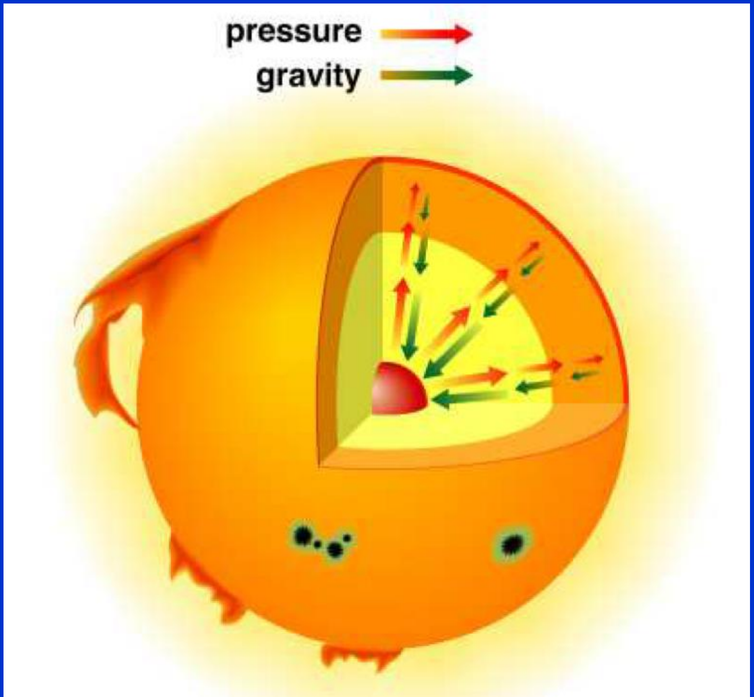
«It is to Eddington more than to any other one man that we owe the rapid development of astrophysics that took place immediately after the work of Planck, Einstein, and Bohr, and we must certainly account him among the giants of that period.»

L. Motz, 1959



«It is not a question of unrestrained conjecture remote from observational facts. The astronomer has any amount of facts to build on, and cannot escape the duty of trying to combine the facts into some sort of order».

«To measure the rate of radiation of a star is to measure its liberation of subatomic energy».



Before Eddington

J.H. Lane (1870): «On the theoretical temperature of the Sun, under the hypothesis of a gaseous mass maintaining its volume by its internal heat»

$$dP = -\rho\, g\, dr$$

$\langle\rho\rangle \sim M/R^3$ $\quad g \sim -GM/R^2$

$$P_c \sim G\, M/R^4$$

$5\ 10^{15}\ g\ s^{-2}\ cm^{-1}$

If perfect gas $\quad P = (k/\mu\, m_H)\, \rho\, T$

With P_c and $\langle\rho\rangle$ $\quad T_c \sim (\mu\, m_H/k)\ (GM/R)$ $\quad \sim 10^7$ °K

NOT BELEIVED AT THE TIME

Solar density = 1. 41 g/cm^3 : atoms touch each other

Heating by contraction

DEEP INVESTMENT IN PHYSICS: QUANTUM PHYSICS

Einstein equation - Black body radiation (Planck law)

Atomic levels of energy → Line formation → stellar spectra

RADIATION (Instead of convection currents)

(Studies by Emden 1907 and by K. Schwarzschild, 1906, 1916 and 1917.)

T-gradient in Capella 1°K /km: **LTE**

Radiation pressure $P_{rad} = (1/3)\ a\ T^4$ $P_{tot} = P_{gas} + P_{rad}$

The outward flowing radiation may thus be compared to a wind blowing through the star and helping to distend it against gravity.

Radiative flux ←→ transfer of momentum

Flux ~ dP_{rad}/dr , inv. prop. to the obstruction ~$1/\kappa\rho$

$dP_{rad}/dr = -\kappa\rho\ F/c$ ⟹

$$\mathrm{Frad} = -\left(\frac{4\,a\,c\,T^3}{3\kappa\rho}\right)\frac{dT}{dr}$$

«....it may be used without hesitation.» → L ~ M^3

We can imagine a physicist on a cloud-bound planet who has never heard tell of the stars calculating the ratio of radiation pressure to gas pressure for a series of globes of gas of various sizes, starting, say, with a globe of mass 10 gm., then 100 gm., 1000 gm., and so on, so that his nth globe contains 10^n gm.

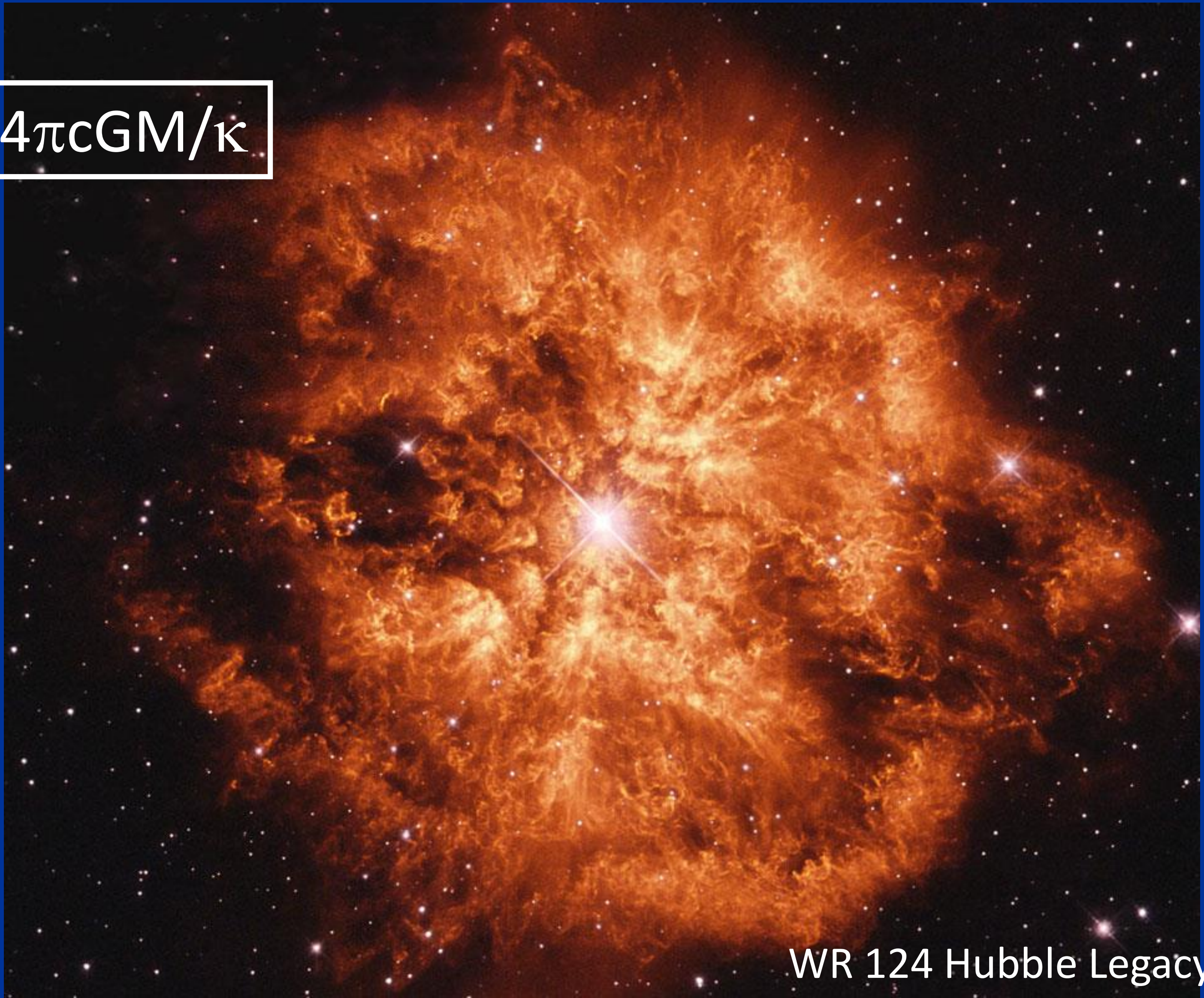
n	Prad/P	Pgas/P
…	….	…
32	0.0016	0.9984
33 1Ms	0.106	0.894
34 10Ms	0.570	0.430
35 100Ms	0.850	0.150
36	0.951	0.049
37	0.984	0.016
38	0.9951	0.0049
39	0.9984	0.0016
….	…	…

What "happens" is the stars.

Why ?

We draw aside the veil of cloud beneath which our physicist has been working and let him look up at the sky. There he will find a thousand million globes of gas nearly all of mass between his 33rd and 35th globes—

The physical explanation of these upper limits is that the radiation observed to be emitted must work its way through the star, and if there were too much obstruction it would blow up the star.



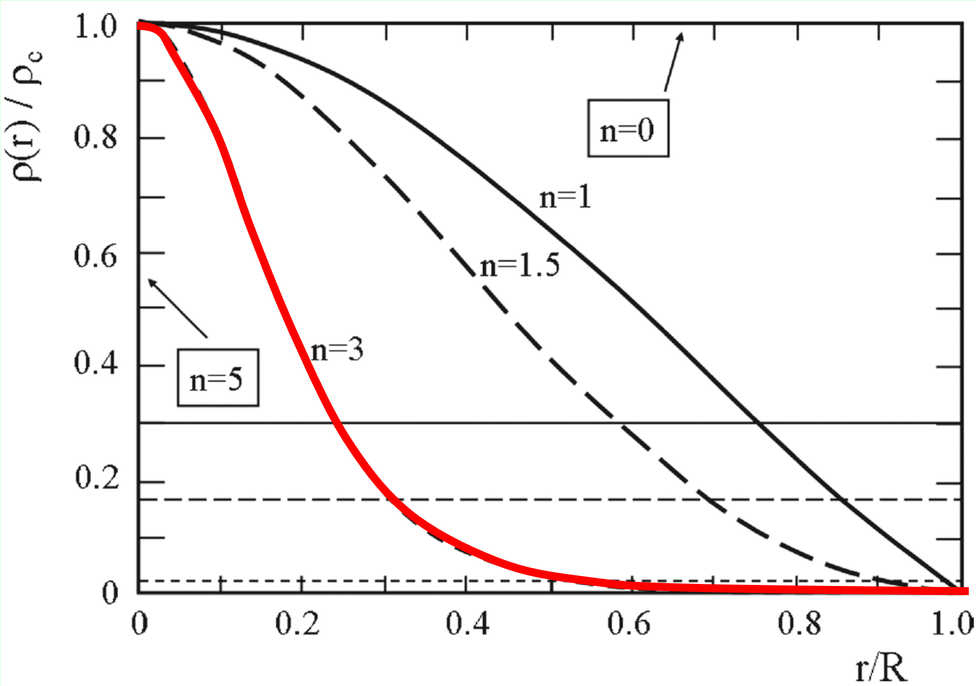
$L_{EDD} = 4\pi cGM/\kappa$

STAR MODELS Emden, 1907: Gaseous Spheres).

Stars with $P = P_{gas} + P_{rad}$ are polytropes $P = k\,\rho^n$

($n=4/3$, if $\beta = P_{gas}/P_{rad}$ constant)

P, ρ, T known even if energy source unknown



STABILITY (Russell, 1919): If energy is added, the star expands and cools and may restaure **equilibrium.** Specific heat negative !

RELATIONS BETWEEN M, L, R, T_c, ρ_c, β,...

M-L relation $L \sim M^{7/5} (1-\beta)^{3/2} \mu^{4/5} T_{eff}^{4/5}$

no ε !

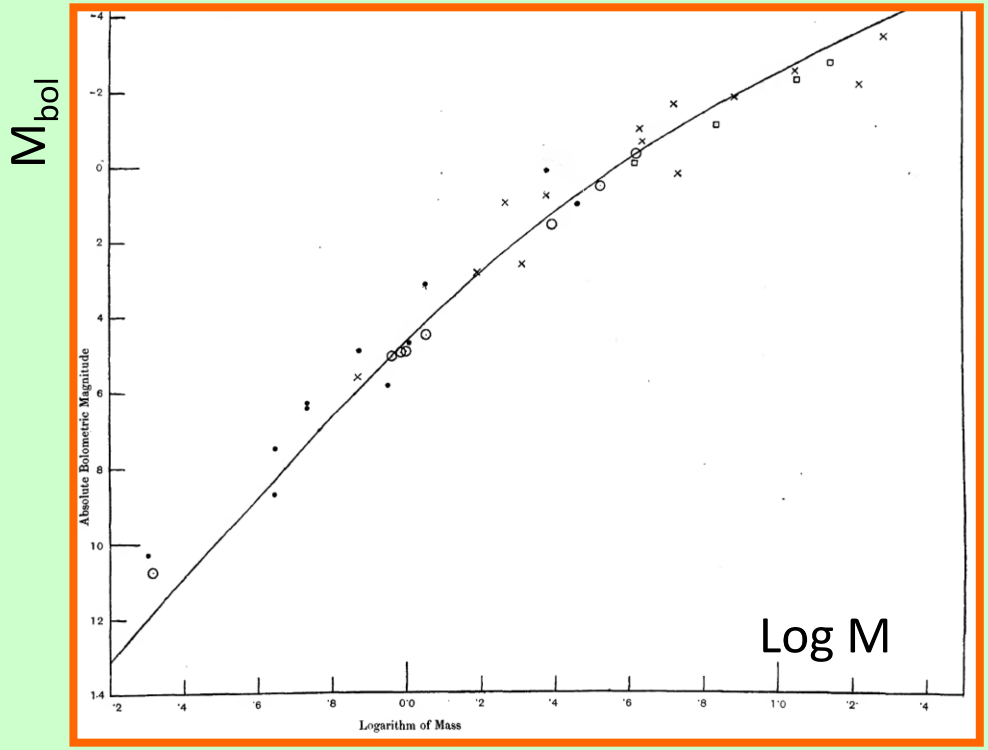
Eddington, 1924

The M – L relation (1924)

Depends on μ, κ (Capella)

Data from binaries (Hertzsprung , 1923), Eclispsing binaries (Shapley, Plaskett). Cepheid pulsations.

Most first class data are dwarfs which fit the perfect gas curve.



SURPRISE !

Sun should have been 3-4 magn below, 9 magn below for Kruger 60 (dev. from p.gas)

THE DWARFS OBEY THE PERFECT GAS LAW

IONISATION: atoms → nuclei (volume 10^6 smaller)

Saha 1921-1925 → T of stars. Equation of ionisation equilibrium.

If ionized → **PERFECT GAS** $P = (\mathcal{R}/\mu)\ T$ μ : much smaller

Eddington: electric effects ions- free electrons → lower P (-0.015 in the Sun)

The source of stellar energy

When discussing this problem, we could easily extract here or there a few sentences from the 30 pages that Eddington devoted to the subject and show how they well fit with what we know today. Indeed, this would not well reflect the complex reality, and not show the extreme difficulty of disentangling contradictory observations and theories until a proper understanding of the problem emerged. In this context the merit of Eddington is immense. He clearly set the problem, without hiding any difficulties. He mentioned "a critic might count up a large number of fatal objections". In the History of Science, Chapter XI of Eddington's book gives a beautiful example of Science in the making, when people search the true solution among a Capernaum of observations, claims, objections and theories. Science in the making is very different from well established Science in the textbooks.

L. Motz 1959

ENERGY SOURCES

$L_O = 3.845 \ 10^{26}$ W, $M_O = 1.989 \ 10^{30}$ kg

Complex problem – immense merit **«…a critic could count up a number of fatal objections»** - **Science in the making !**

Contraction: Kelvin, **Helmholtz:** energy liberated $(3/2)\,(GM^2/R)$
→ supply L_O for < 20 million yr. Clash with geological constraints.
Also, if contraction, $\Delta R/R$: 1/40 000 yr^{-1} → δ Cephei ΔP: (17 s yr^{-1})

External source:

203. In seeking a source of energy other than contraction the first question is whether the energy to be radiated in future is now hidden in the star or whether it is being picked up continuously from outside.

The T-gradient cannot be maintained by supplying energy at the cool side

Physics:

The difficulty is that from the physicist's point of view the temperature of the stars is absurdly low. He regards the stars as practically at absolute zero, because in regard to nuclear processes 40 million degrees is a small quantity which it is scarcely worth while to take notice of. If liberation of subatomic energy occurs freely on the stars, why not on the earth?

Astronomical difficulties:

1. Capella liberates 58 ergs per gram per second compared with 1·9 liberated by the sun.

2. The density of the sun is 620 times the density of Capella.

3. The temperature of the sun at corresponding points is 4·3 times the temperature of Capella.

Why then is there this decreased output in the sun in spite of the apparently more favourable conditions?

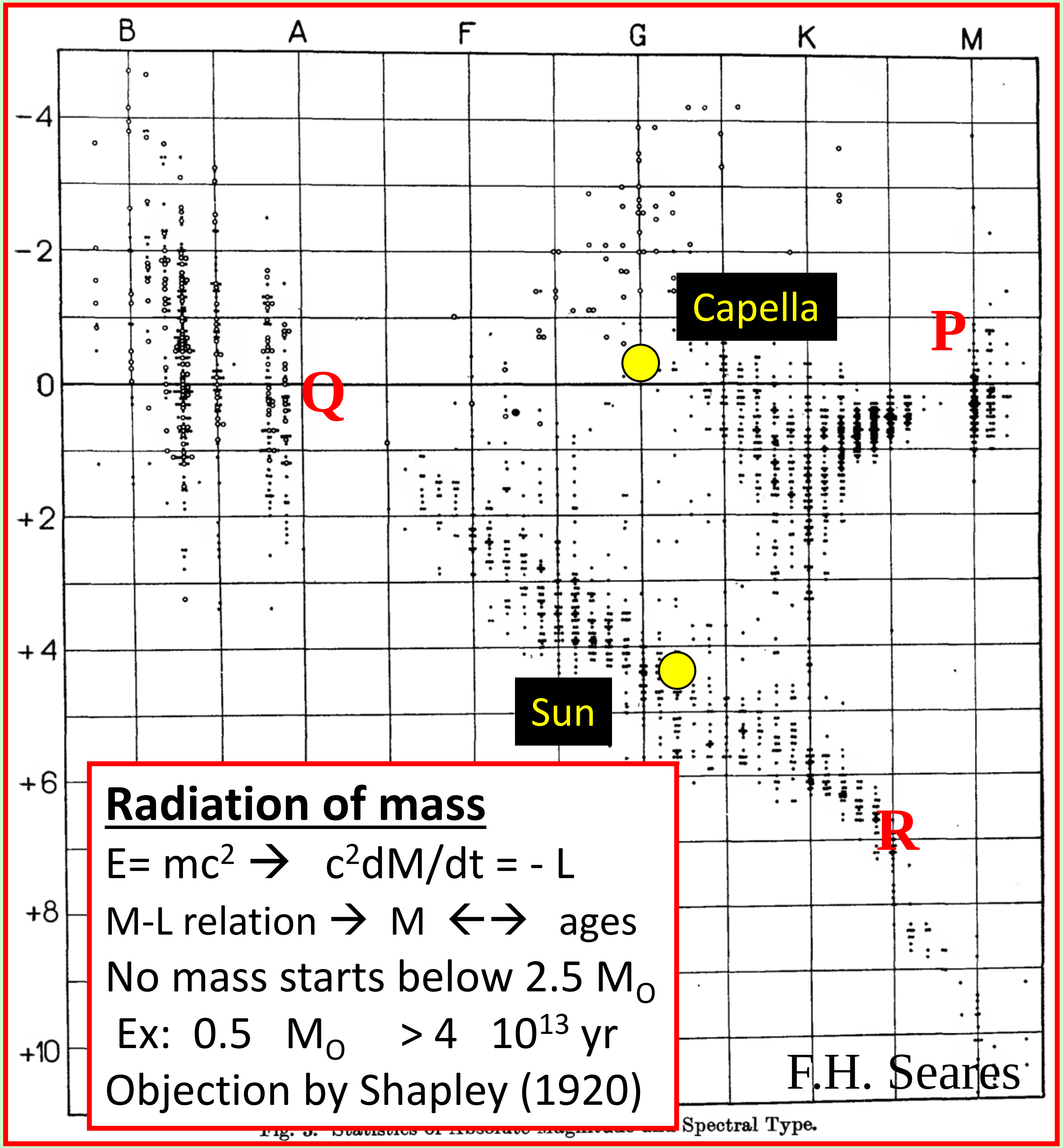
Radioactivity:

So far as we know, the processes I (*a*) give much less energy.

Stars would be unstable! ε must increase with ρ and T .

Annihilation, radiation of mass

Mutual cancellation of electrons and protons?



Radiation of mass

$E = mc^2 \rightarrow \quad c^2 dM/dt = -L$

M-L relation → M ←→ ages

No mass starts below 2.5 M_O

Ex: 0.5 M_O > 4 10^{13} yr

Objection by Shapley (1920)

Two series:

Giants – Dwarfs

Capella:

$<\rho> = 0.0023$ g/cm^3

Perfect gas

Sun: 1.411 g/cm^3

Liquid ? (trans. 0.1-0.5)

EVOLUTION

(Hertzsprung, Russell)

P → Q → R

OK with radiation of mass.

SUBATOMIC ENERGY

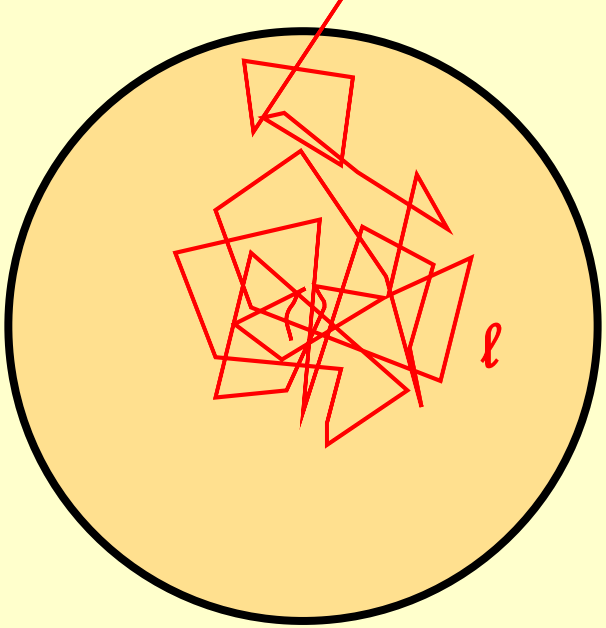
F. W. Aston (1920): mass-spectrograph

Formation of He from 4 H: a loss of mass of 0.8 %. **Lot of energy.**

Eddington (1926) sums up the reasons for and against:

- **Only process providing sufficient energy.**
- «unless the initial proportion of H in a star is unduly large the length of the life of the star is barely sufficient».
- Comparison with Capella: «the sun must very nearly have exhausted its stock of hydrogen and its future life will be short».
  Especially more than there is very little energy release after He.
- Advanced elements (Ti, Zr,…) in early stars and He, C in diffuse nebulae «strongly against admitting a large proportion of H in early stars»
- Process unknown, **but it has occurred**.

In particular the change of mass is insignificant; and unless the star can gain or lose mass from other causes there is no evolution from bright to faint classes of stars.



OPACITY of the stellar material

If direct from the center: $t = 2.3$ sec
With many absorptions – emissions

$t \sim (\ell/c)\ N$ $N = 3\,(R/\ell)^2$
$\ell \sim 1/(\kappa\,\rho)$ $t \sim 30$ millions yr.

κ taken constant before 1922

Kramers' quantum theory of emission → $\kappa \sim (\rho/\mu)\ T^{7/2}$

κ_a : from observed masses and luminosities

κ_t : Kramers theory

Composition: ~terrestrial

$\kappa_a = 10\ \kappa_t$ 1923

By assuming that the stars contain a large proportion of hydrogen the discrepancy would be removed;

«I would much prefer to find some other explanation of the discordance between κ_t and κ_a.»

High H-abundance: Unsöld 1928, McCrea 1929, Russell 1929; **Strömgren 1932 (33%)**

Lots of heavy debates.

With Milne who studies the photosphere. Eddington, 1930, MNRAS, 90, 284.

I do not think he has realised how disconnected is his field of investigation from the problems of the interior.

Eddington, 1930, Observatory, 53, 208)

I almost despair of extricating my main controversy with Prof. Milne from the tangle in which it has become involved. I would like therefore to call attention to one point of difference which may perhaps explain why he proclaims ignorance where I claim a substantial advance of knowledge.

About the surface boundary conditions (cited by Wali, 1990): *I have not read Professor Milne's paper, but I hardly think it is necessary, for it would be absurd for me to pretend that Professor Milne has the remotest chance of being right.*

E.A.Milne, 1930 MNRAS 90, 67

In his first paper Eddington replies to an accusation I did not make, and in his second paper he makes a serious mistake in reversing some of my algebra, and so overlooks my main theorem.

WHITE DWARFS

Sirius B: ~0.85 M_O, (Clark 1862) W.S. Adams 1914: type A-F (8000K)
M_V =11.3 → R=18 800 km, ρ = 61 000 kg/dm³

This argument has been known for some years. I think it has generally been considered proper to add the conclusion "which is absurd."

Crucial test: predicted Einstein gravitational shift	= 20 km/s	
Test by Adams at Mt Wilson (1925): mean obs. radial vel.	= 23 km/s	Sirius B
Orbital motion	-4.3 km/s	
Obs. Einstein shift	= 19 km/s	

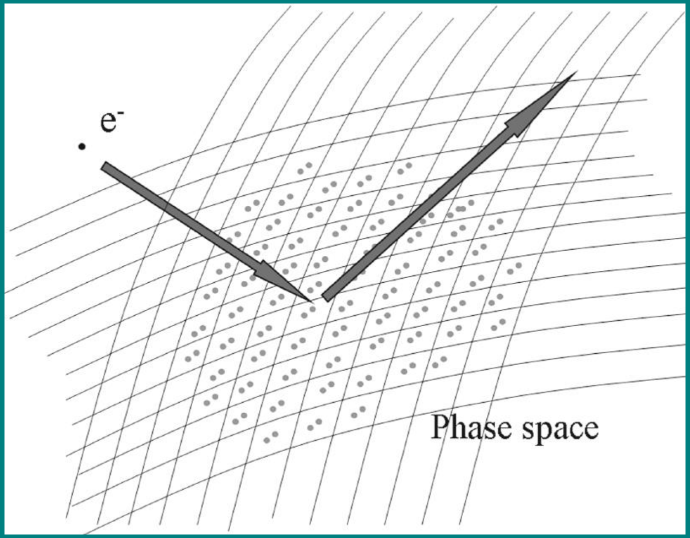
Prof. Adams has killed two birds with one stone; he has carried out a new test of Einstein's general theory of relativity and he has confirmed our suspicion that matter 2000 times denser than platinum is not only possible, but is actually present in the universe.

One of the first 3 tests of the GR.

PHYSICS OF WD

Pauli Exclusion Principle : the extension in phase space obeys

$$\Delta^3 q_i \; \Delta^3 p_i \; > \; h^3$$



Electrons reach this limit first,
obey Fermi-Dirac statistics.

$$P = K_1 \; (\rho/\mu_e)^{5/3}$$

For full degeneracy

(metals at terrestrial conditions)

R.H. Fowler (1926)

Chandrasekhar (1931):

Higher ρ (> $4\ 10^6$ g/cm^3), e^- relativistic.

$$P = K_2 \; (\rho/\mu_e)^{4/3}$$

$GM^2/R^4 \sim (K_2/\mu_e^{4/3})\,(M^{4/3}/R^4)$ → $M_{Ch} \sim 1.46\ M_O$

Strongly criticized by Eddington

Max. mass for a WD

Eddington (1935):

When its supply of subatomic energy is exhausted, the star must continue radiating energy and therefore contracting—presumably until, at a diameter of a few kilometres, its gravitation becomes strong enough to prevent the escape of radiation. This result seems to me almost a *reductio ad absurdum* of the relativistic formula. It must at least rouse suspicion as to the soundness of its foundation.

I do not think that any flaw can be found in the usual mathematical derivation of the formula. But its physical foundation does not inspire confidence, since it is a combination of relativistic mechanics with non-relativistic quantum theory.

In the present paper this unholy alliance is examined. The conclusion is reached (§ 5) that the "relativistic" formula is erroneous, and that the correct formula is $P_e = K\sigma^{\frac{5}{3}}$.

1935: IAU General Assembly in Paris. Clash (Eddington-Chandra.)
1939: Meeting in Paris. Kuiper shows obs. in favour of Chandra's theory.
1979: Back to IAU General Assembly in Montreal.

K.C. WALI, 1990, Chandra, A biography of S. Chandrasekhar.
A.I. MILLER, 2005, Empire of stars: Obsession, Friendship, and Betrayal in the Quest for Black Holes.
G. SHAVIV, 2013, The discovery of the Chandrasekhar mass and the Chandrasekhar-Eddington controversy.

The moral: Eminent scientists are not immune against making colossal mistakes and perusal biases.

Edmund C. Stoner (Univ. Leeds) found the max. mass of WD in 1931 (one year before Chandra.)

The positive conclusion about Eddington:

See S. Chandrasekhar, *Eddington: the Most Distinguished Astrophysicist of his Time*, Cambridge Univ. Press, 1983, for an overall account of his contributions to physics.

EDDINGTON

The most distinguished astrophysicist of his time

S. CHANDRASEKHAR

University of Chicago

CAMBRIDGE UNIVERSITY PRESS

Cambridge

London New York New Rochelle

Melbourne Sydney

1983

VARIABLE STARS

170 Cepheids known (P from hours to 50 days)

Relation magnitude-period (H. Leavitt 1912).

If binaries, secondaries would be inside the principal (Shapley 1914)

Eddington considers perturbation of

$(dp/dr) = -\rho g - \rho(d^2r/dt^2)$ → Diff. Equ. for perturbations

with $P = \rho^\gamma$ adiabatic $\Pi \, \rho_c^{1/2} \approx$ const.

Dissipation by heat leakage → limits pulsation

$d\mathcal{T}\nearrow = \oint (\Delta T/T_o)\ \delta Q \swarrow$

Work provided by pulsation

Pulsation: - if heat provided when hotter
- if loss of heat when cooler

ε mechanism: ε grows with T↗ (compr., T and ε↗, expansion, gravity recall)

κ mechanism: κ↘ with T↗, more heat leakage when hotter → STABLE

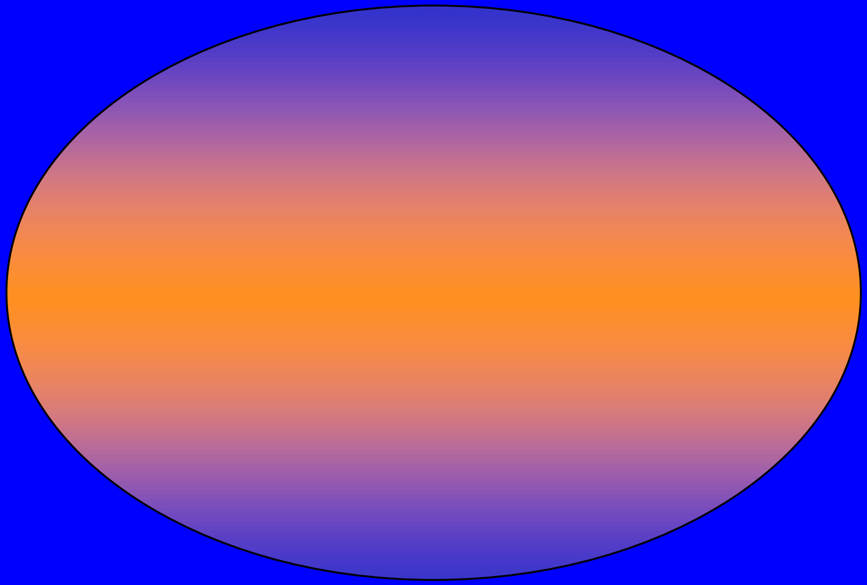
But at level where ionisation of predominant elements occurs, the behaviour of vs. T might change.

ROTATION If Ω=const. $(1/\rho)\,\nabla P = \nabla\,\Phi + \nabla V = \nabla \Psi$

Equipotentials are isobars : $P=P(\Psi)$, $\rho=\rho(\Psi)$, $T=T(\Psi)$.

$F_{rad} \sim \nabla T \sim \nabla \Psi$

$$F_{rad} \sim g_{eff}$$



Von Zeipel (1924)
Confirmed by VLTI observations
(Peterson, 2006; Monnier, 2007)

Equipotentials are closer at the pole than at the equator →
Excess of flux → thermal imbalance → circulation current →
MIXING Studies in 1925, 1926, 1929 (< 60 m per yr).

SIGNIFICANT EFFECT IN EVOLUTION (WITH SHEARS)

Radiation of mass

$E = mc^2$ implies $dM/dt = -L/c^2$

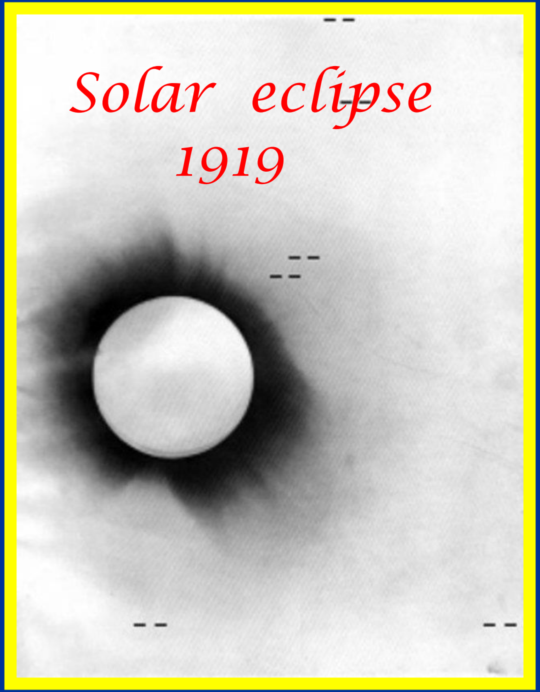
M-L relation → masses ←→ ages

Assumed no mass starts below 2.5 M_O → low masses are very old.

Ex: 0.5 M_O > 4 10^{13} yr

Eddington: «the fainter stars are missing because the cluster has not existed for a sufficient time to evolve them».

Objection by Shapley (1920): Praesepe cluster contains giants → young, the numerous dwarfs cannot have evolved by radiation of mass, if stars are coeval in a cluster.



«It is to Eddington more than to any other one man that we owe the rapid development of astrophysics that took place immediately after the work of Planck, Einstein, and Bohr, and we must certainly account him among the giants of that period.» *L. Motz, 1959*

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Eddington (1939)

A white dwarf stage thus precedes the main series stage; but, if the time-scale permits, the star may return a second time to the white dwarf stage after it has exhausted its hydrogen.